Before Topic 5 – Summary of the "TRANSFORMATIONS" studied so far and more….

(1) Fourier Series of a periodic signal in continuous time.

It is also called

Continuous Time Fourier Series (CTFS)

(2) Fourier Transform of *a signal with finite energy* in continuous time.

It is also called

Continuous Time Fourier Transform (CTFT)

(3) Fourier Series of a periodic signal in discrete time.

It is also called

Discrete Time Fourier Series (DTFS)

(4) Fourier Transform of *a signal with finite energy* in discrete time.

It is also called

Discrete Time Fourier Transform (DTFT)

(5) Generalized Fourier Transform (GFT) of *a periodic signal* in continuous time.

It can be also applied for a signal z(t) defined as a sum of periodic signals, where z(t) is not periodic.

(6) Generalized Fourier Transform (GFT) of *a periodic signal* in discrete time.

It can be also applied for a signal z[n] defined as a sum of periodic signals, where z[n] is not periodic.

Summary: “Fourier so far”

	Periódica en el tiempo	No periódica en el tiempo	
Continua en el tiempo	**CTFS** Analysis $a_k = X[k] = \frac{1}{T}\int_{<T>} x(t)\, e^{-jk\omega_0 t}\, dt$ Synthesis – CT Fourier Series $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	**CTFT** Synthesis – Inverse Fourier Transform $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t}\, d\omega$ Analysis – CT Fourier Transform $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt$	**No periódica en frecuencia**
Discreta en el tiempo	**DTFS** Synthesis – DT Fourier Series $x[n] = \sum_{k=<N>} a_k e^{jk\Omega_0 n}$ Analysis $a_k = X[k] = \frac{1}{N}\sum_{n=<N>} x[n] e^{-jk\Omega_0 n}$	**DTFT** Synthesis – Inverse Fourier Transform $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega) e^{j\Omega n}\, d\Omega$ Analysis – DT Fourier Transform $X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$	**Periódica en frecuencia**
	Discreta en frecuencia	**Continua en frecuencia**	

- did you have studied something more?

Generalized Fourier Transforms (mathematically…is …)

- for periodic signals (including constants)

$$a_k = X[k] = \frac{1}{T}\int_{<T>} x(t)e^{-jk\omega_0 t}dt$$

$$\omega_0 = \frac{2\pi}{T}$$

$$X_G(\omega) = 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$$

$$X_G(\omega) = 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - k\frac{2\pi}{T}\right)$$

$$a_k = X[k] = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\Omega_0 n}$$

$$\Omega_0 = \frac{2\pi}{N}$$

$$X_G(\Omega) = 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\Omega - k\Omega_0)$$

$$X_G(\Omega) = 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\Omega - k\frac{2\pi}{N}\right)$$

Generalized Fourier Transforms (mathematically…is …)

- The Generalized Fourier Transform can be also used for SUM OF PERIODIC SIGNALS
- even if the signal defined as sum of signals is not periodic

$$\underset{\text{non-periodic}}{z(t)} = \underset{\text{periodic}}{x_1(t)} + \underset{\text{periodic}}{x_2(t)}$$

$$\boxed{Z_G(\omega) = X_{1G}(\omega) + X_{2G}(\omega)}$$

- does z(t) admit Fourier Series? NO!

(7) Discrete Fourier Transform (DFT)

Very similar to the DTFS (i.e., Fourier Series of a periodic signal in discrete time).

The difference is just a multiplication factor 1/N.

It is used by Matlab (or similar) to compute/approximate the Fourier Transform/series.

(8) Fast Fourier Transform (FFT)

It is the same of DFT (just faster)

FFT=DFT (!!!)

It is used by Matlab (or similar) to compute/approximate the Fourier Transform/series.

(9) Laplace Transform (LT)

It is a generalization of CTFT (i.e., Fourier Transform of a signal with finite energy in continuous time)

We will study (Topic 5):

(10) Zeta Transform (ZT)

It is a generalization of DTFT (i.e., Fourier Transform of a signal with finite energy in discrete time)

Extension: Laplace and Zeta transforms

LAPLACE TRANSFORM

$$X(s) = \int_{-\infty}^{+\infty} x(t) e^{-st} dt$$

$$s = \sigma + j\omega$$

$$X(\omega) = X(s)\Big|_{s=j\omega}$$

$$\sigma = 0$$

ZETA TRANSFORM

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n] z^{-n}$$

$$z = re^{j\Omega}$$

$$X(\Omega) = X(z)\Big|_{z=e^{j\Omega}}$$

$$r = 1$$

SUMMARY of Acronyms

CTFS ------ CTFT
DTFS ------ DTFT

GFT

DFT = FFT

LT ------ ZT

Do you know all of them?
Possible questions of the exam:
"What is the meaning of FFT ? "
"What is the meaning of DTFS ? "