Topic 2 - Part 1: SYSTEMS IN THE TIME DOMAIN

Linear systems and circuit applications

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Based also on Professor Óscar Barquero Perez, Andrés Martínez and José Luis Rojo's slides

Introduction: what is a system?

- **Any transformation, any mapping of a signal into other signal:**

$$\boxed{y(t) = F\{x(t)\}}$$

$$y(t) = f(x(t), x(t-1), x(t+2)....)$$

- **even with feedback (autoregressive systems):**

$$y(t) = f(x(t), x(t-1), ..., y(t-1), y(t-2))$$

Introduction: what is a system?

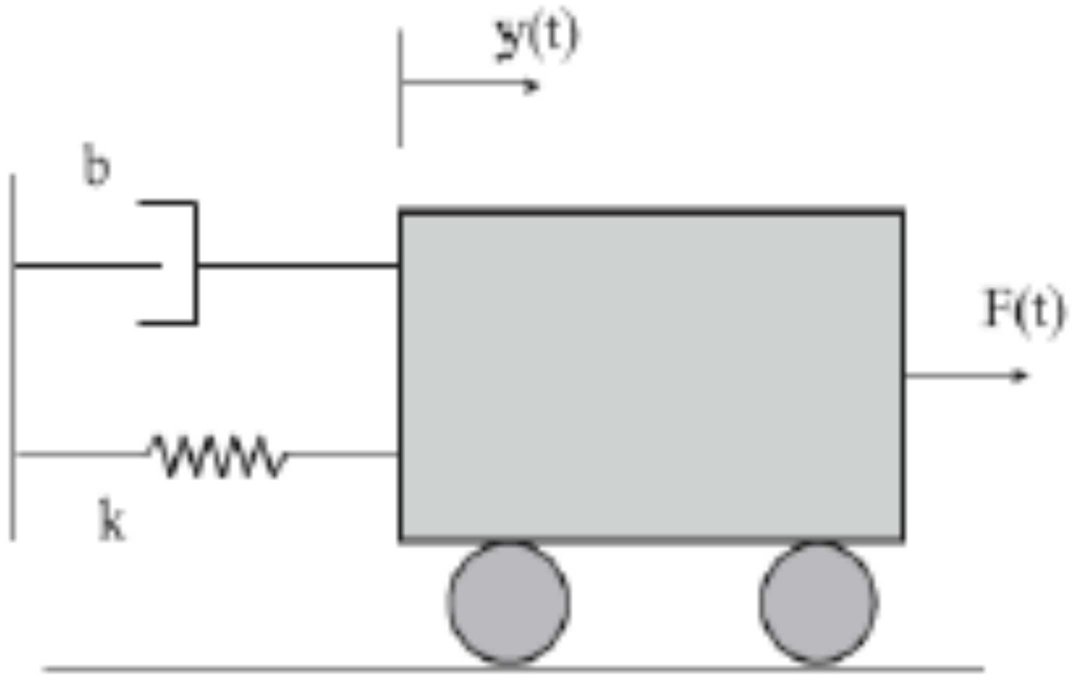
System definition

- A **system** can be viewed as a process in which input signals are transformed by the system or cause the system to respond in some way, resulting in other signals as outputs.

Example: dynamical systems

- A simple dynamical system: a little car on a surface, tide to the wall by a spring.
- Law of forces:

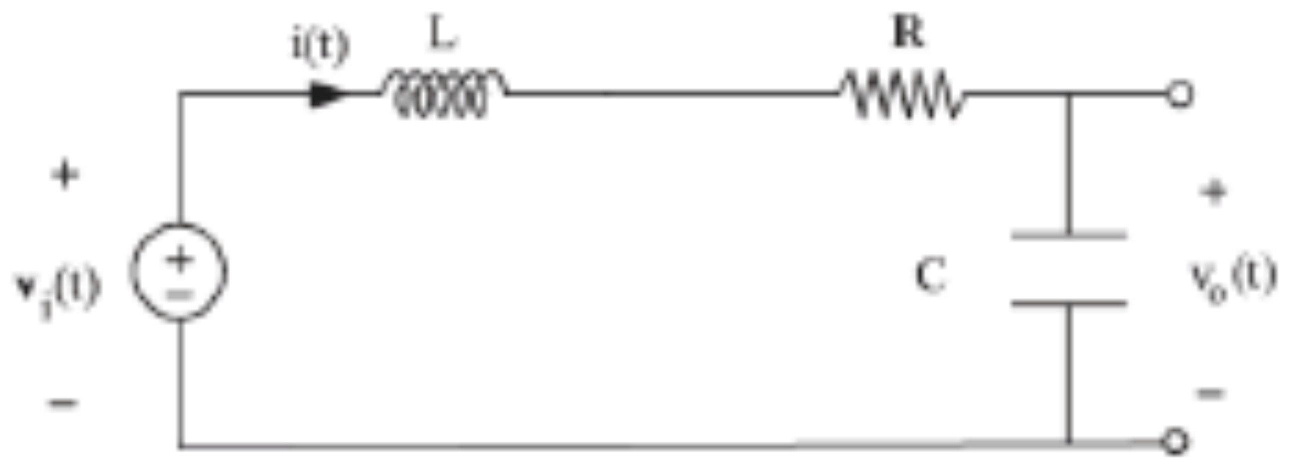
$$M\frac{d^2y(t)}{dt^2} + b\frac{dy(t)}{dt} + ky(t) = F(t)$$



Introduction: what is a system?

Example: a circuit system

- RLC circuit. The input is $v_i(t)$, an arbitrary signal.
- The output $v_o(t)$ will be a transformation of the input.
  Is there a equation relating them?



$$LC\frac{d^2v_0(t)}{dt^2} + RC\frac{dv_0(t)}{dt} + v_0(t) = v_i(t)$$

- It is a second order diferential equation. Note the similarity with the mechanical system..
- The signal and systems tools can be used in many applications.

Introduction: what is a system?

Example: Integrator Systems

- We have an integrator system, which input is the signal $x(t) = tu(t)$. Therefore, for $t < 0$:

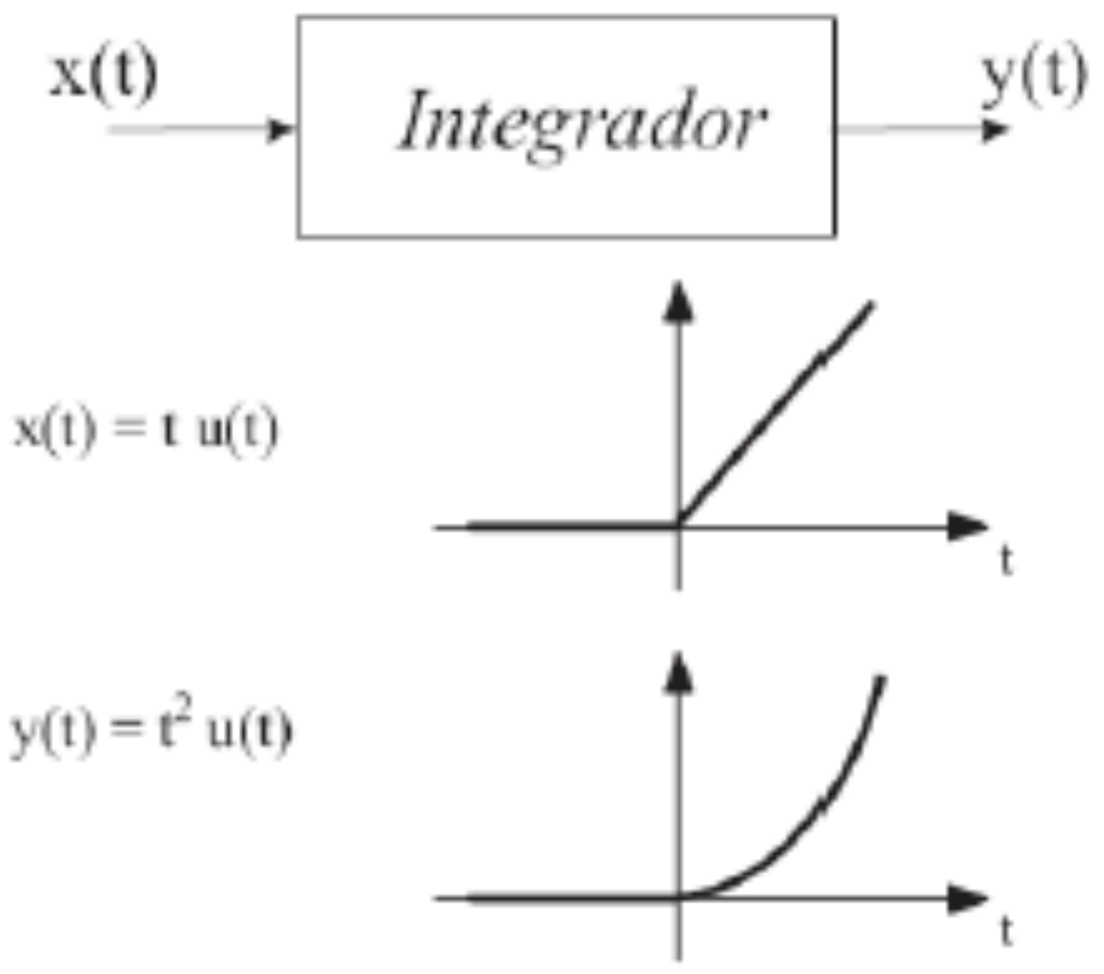
$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau = \int_{-\infty}^{t} 0 d\tau = 0$$

whereas for $t \geq 0$:

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau = \int_{0}^{t} \tau d\tau = \left[\frac{\tau^2}{2}\right]_0^t = \frac{t^2}{2}$$

- The output can be expressed using the unit step signal:

$$y(t) = \frac{1}{2}t^2u(t)$$



Some properties of a system

- In the following, we will describe some properties of a generic system:
- Memory
- Causality
- Stability
- Time invariance
- Linearity

Memory

Memory

- A system is said to be **memoryless** if its output, for each value of t, is dependent only on the input at that same time, that is, $y(t) = f(x(t))$.
    - **A system is "with memory"** ***in any other case*****.**
    - **Note that the memory can be** ***the past or the future*****….**

Examples

- Memoryless systems:
    - $y(t) = (2x(t) - x^2(t))^2$.
    - A resistor, in which $y(t) = Rx(t)$.
- Systems with memory:
    - A delay system, $y(t) = x(t-2)$.
    - A capacitor $v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(\tau)d\tau$.

Memory

(*)Determine if each of the following systems are memoryless or with memory:

1. $y(t) = t \cdot x(t)$. → memoryless
2. $y(t) = x(t+4)$. → with memory
3. $y(t) = \sum_{k=-3}^{0} x(t-k)$. → with memory
4. $y(t) = x(-t)$. → with memory
5. $y(t) = \cos(3t)x(t)$. → memoryless
6. $y(t) = x(t) + 0.5y(t-2)$. → with memory

- Note that the memory can be *the past or the future*….

Memory in a image or 2D signal

- **Bidimensional signal**

memoryless **with memory** **with memory** **with memory**

Memory

$$y(t) = \frac{dx(t)}{dt} \longrightarrow ?$$

Causality

- and also outputs values in the past…

Causality

- A system is causal is the output at any time depends only on values of the input at the present time (same time) and in the past. Such a system is often referred to as being *physically feasibles* or *nonanticipative*.

- just to give an idea:

$$y(t) = f(x(t), x(t-1), x(t-2), ..., y(t-1), y(t-2), ...)$$

- we are talking of any mapping, any generic transformation, $f(...)$ or $f\{...\}$

- remember that "inputs" means also delayed version of the outputs

Non-causal and anti-causal

- **Non-Causal:** the output depends on the past and future *jointly* (and can be also dependent on the present).
- **Anti-Causal:** the output depends on the future (and can be also dependent on the present).
- *in different books, there are different definitions…*

Causal, Anti-Causal, Non-causal: examples

- **Causal system:**

$$y(t) = f(x(t), x(t-1), x(t-2))$$

- **Anti-causal system (depends on the future):**

$$y(t) = f(x(t+2)) \qquad y(t) = f(x(t), x(t+1)) \qquad y(t) = f(x(t), y(t+5))$$

- **Non-causal system (depends on the future and the past):**

$$y(t) = f(x(t-1), x(t+3)) \qquad y(t) = f(x(t), x(t-5), x(t+3))$$

Causality: examples

- The system $y(t) = x(t) - x(t-1)$ is causal.
- The system $y(t) = 2x(t+3)$ is anticausal.
- The system $y(t) = x(t-1) - x(t+3)$ is noncausal.

Causality: examples

1. $y(t) = x(-t).$ → Non-causal (…anti-causal…)
2. $y(t) = x(t) \cdot \cos(t+1)$. → Causal
3. $y(t) = Ax(t).$ → Causal
4. $y(t) = \int_{-\infty}^{t+2} x(\tau)d\tau.$ → Non-causal

Causality: examples

$$y(t) = x(at)$$

↓

Non-causal

$$\forall a \quad \text{with } a \neq 1$$

https://www.youtube.com/watch?v=0TzBSqENELM&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=87
https://www.youtube.com/watch?v=A5SlTkKfUz0&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=88

Causality: examples

$$y(t) = x(t - b)$$

$b \geq 0$

Causal

$b < 0$

Anti-causal

Causal systems and memoryless systems

- Every memoryless system is causal

Stability

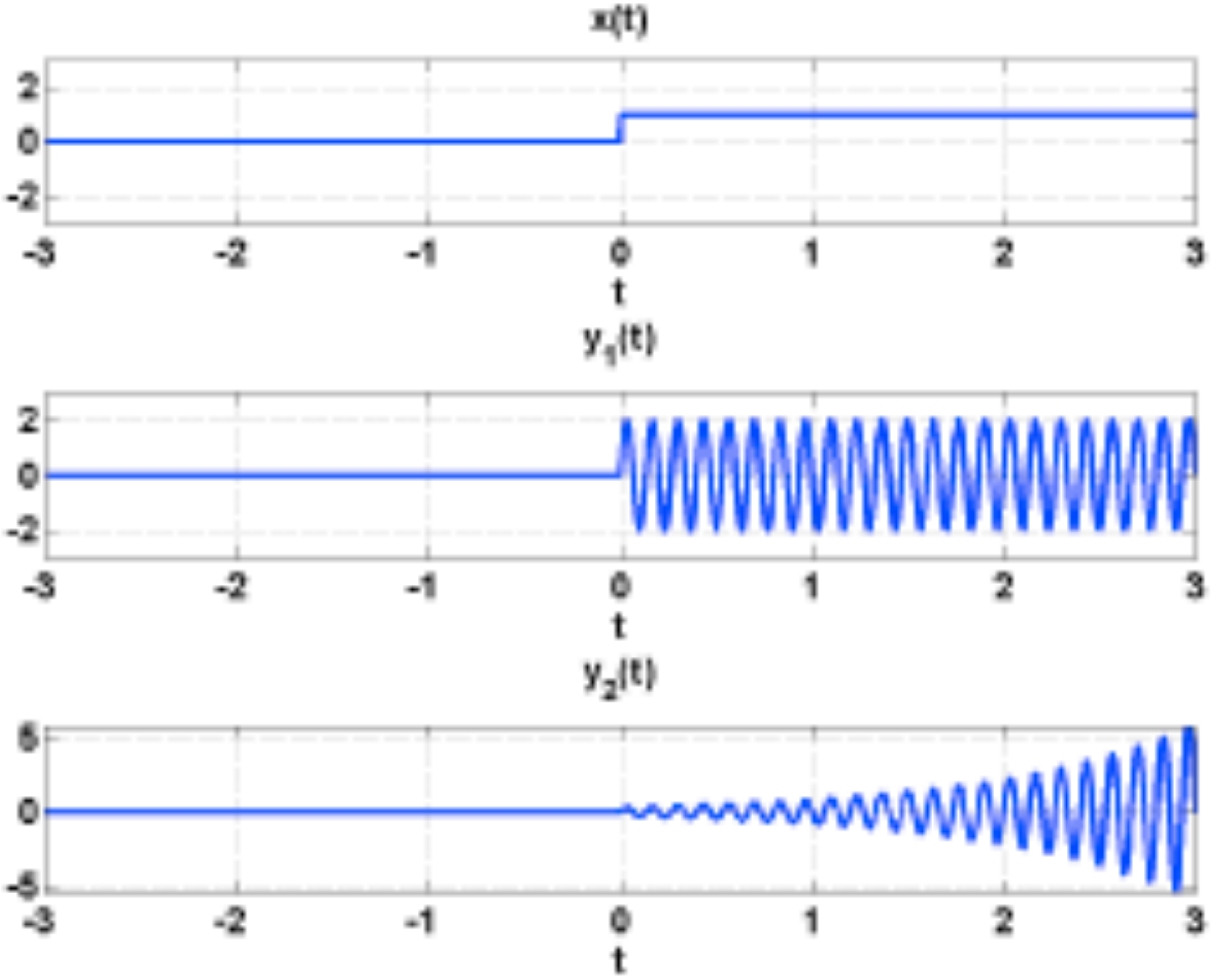
Stability

- A system is sadi to be **stable** when bounded inputs leads to bounded outputs, for any time, t. Mathematically, this property is expressed as (BIBO):

$$|x(t)| < K_x < \infty \Rightarrow |y(t)| < K_y < \infty \qquad \forall t \text{ !!!!}$$

- A system is unstable whenever we are able to find a *specific* bounded input that leads to an unbounded output. Finding one such example enable sus to conclude that the given system is unstable.

Stability



The corresponding system that produces these outputs is:

Stable

Unstable

Stability: examples

1. $y(t) = [x(t)]^2.$ → **Stable**
2. Derivative system: $y(t) = \frac{dx(t)}{dt}.$ → **Stable**
3. Integrator system: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau.$ → **Unstable**
4. $y(t) = t \cdot x(t).$ → **Unstable**
5. $y(t) = x(-t)$ → **Stable**
6. $y(t) = x(t-2) + 3x(t+2).$ → **Stable**
7. $y(t) = Impar\,(x(t)).$ → **Stable**
8. $y(t) = e^{x(t)}.$ → **Stable**

Stability: examples

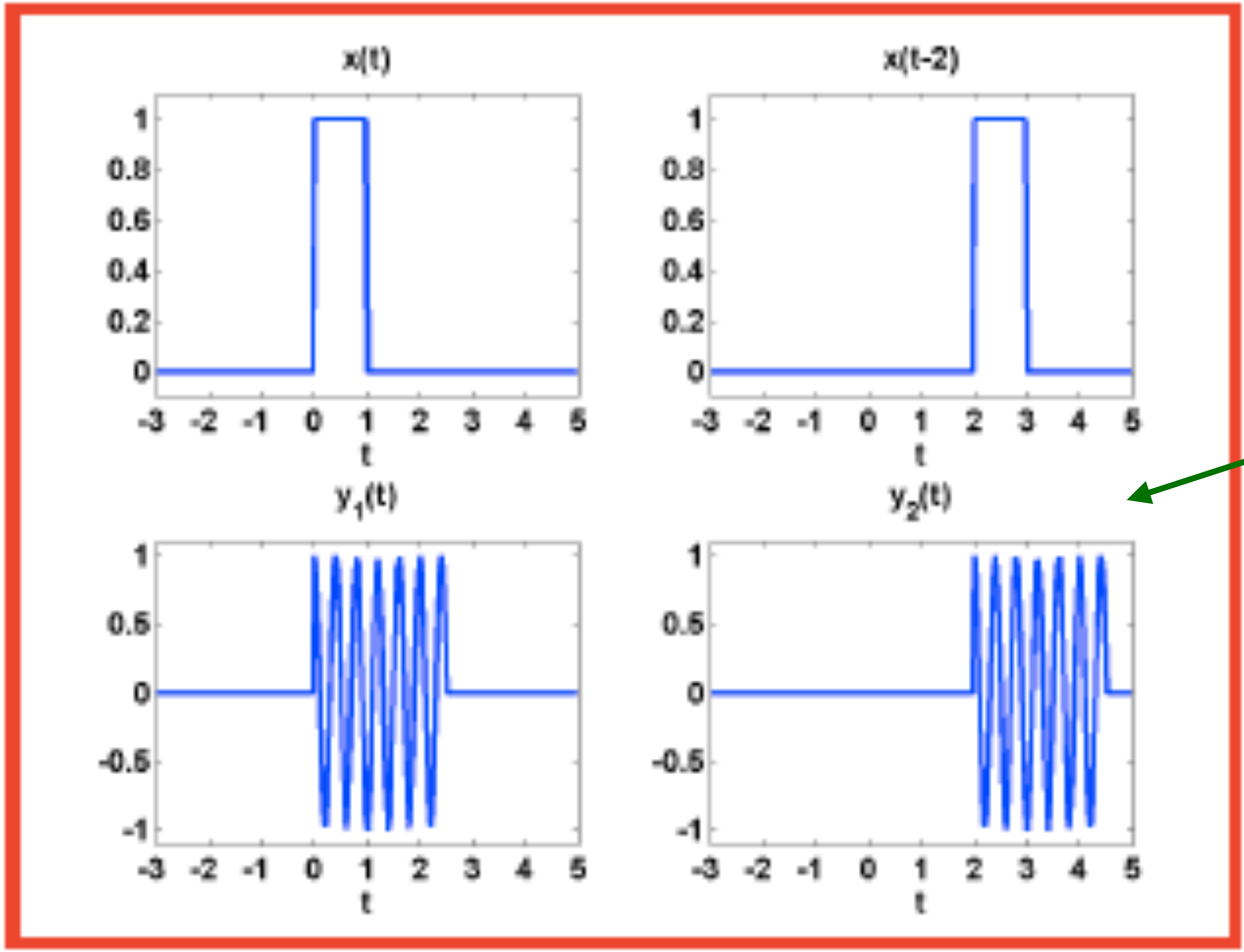
$$y(t) = \frac{1}{x(t) + 1}$$ → **Unstable**

Time invariance

Time Invariance (I)

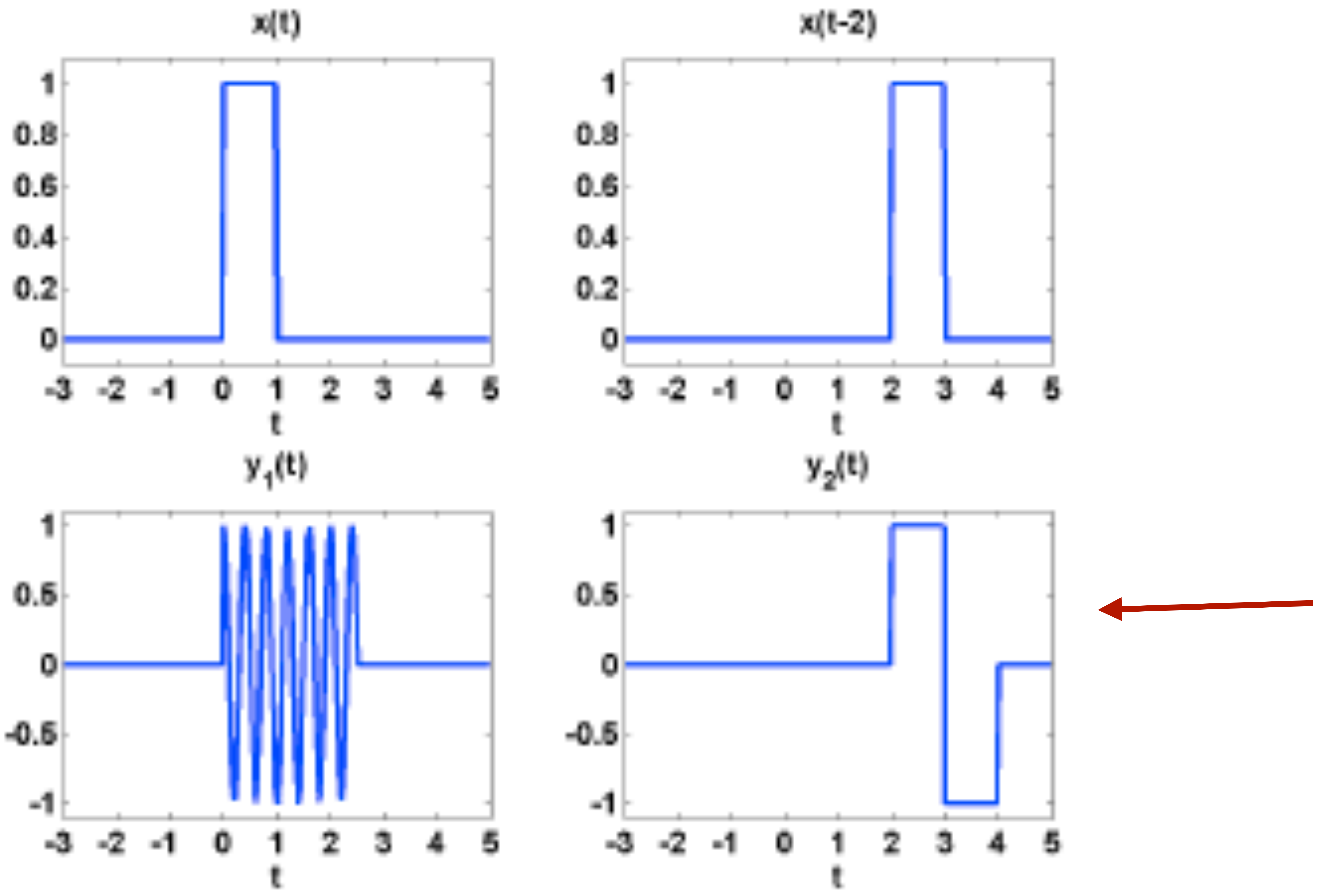
- A system is **time invariant** if the behavior and characteristicas of the system are fixed over time.
- A system is time invariant if a time shift in the input signal results in an identical time shift in the output signal.
- The system is said to be *time variant* otherwise.

Time invariance



Output of a time invariant

Time invariance



Output of a

Time variant system

Time invariance: method

1. Let be $x_1(t)$ an arbitrary input, and let be $y_1(t)$ the output for this particular input.
2. The output is shifted by a given t_0, $y_1(t-t_0)$.
3. Then, consider a second input, $x_2(t)$, which is obtained by shifting $x_1(t)$ in time, $x_2(t) = x_1(t-t_0)$. The corresponding output is $y_2(t)$.
4. We have to compare both outputs $y_2(t) \stackrel{?}{=} y_1(t-t_0)$, if the equality holds, then the system is time invariant.

Time invariance: examples

1. $y(t) = \cos[x(t)].$ → time invariant
2. $y(t) = t + x(t).$ → time variant
3. $y(t) = tx(t).$ → time variant
4. $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau.$ → time variant
5. $y(t) = \frac{dx(t)}{dt}.$ → time invariant

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$ → time invariant

https://www.youtube.com/watch?v=BZq7j2b-7Lw
https://www.youtube.com/watch?v=P4_iWrawCZs

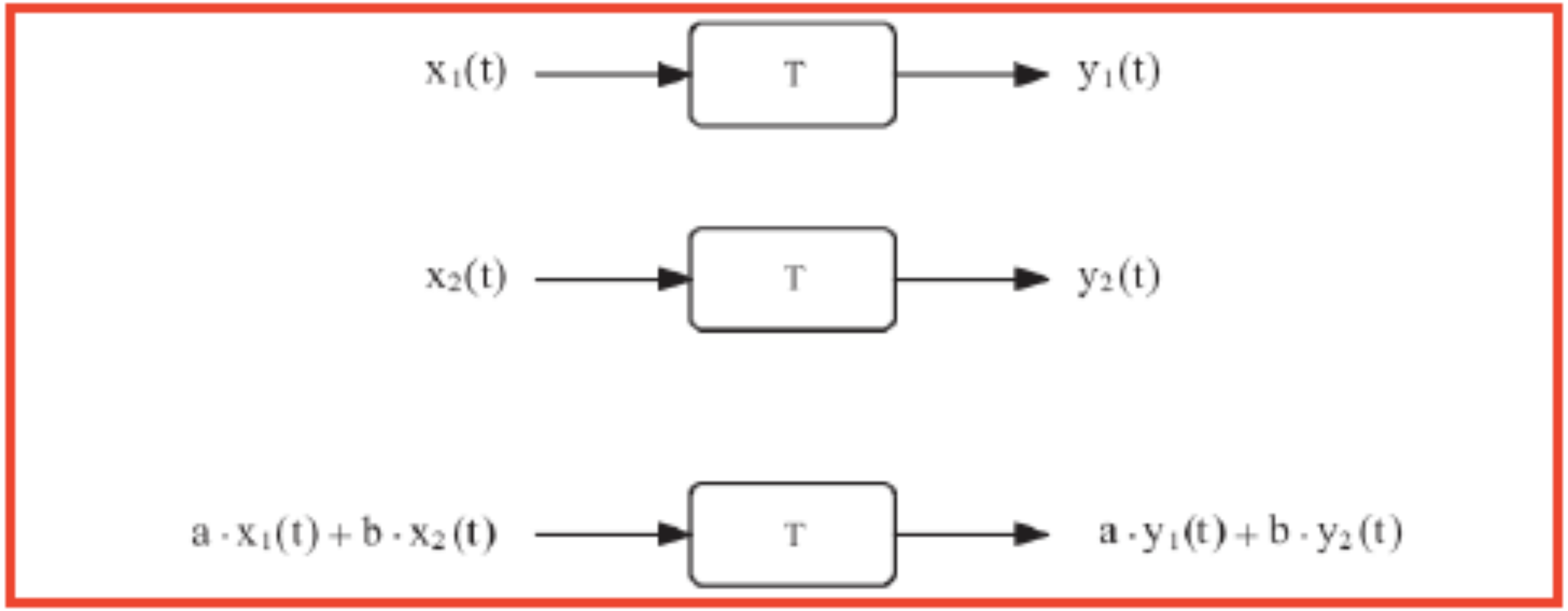
Linearity

Linearity

- A systems is said to be **linear** when it possesse the property of superposition. The property of superposition has two properties: *additivity* and *scaling* or *homogeneity*:
  1. Additivity: the response to $x_1(t) + x_2(t)$ is $y_1(t) + y_2(t)$.
  2. Scaling: the response to $ax_1(t)$ is $ay_1(t)$ (whit $a \in \mathbb{C}$).
- The two properties can be combined. A system is linear when the response to $ax_1(t) + bx_2(t)$ is $ay_1(t) + by_2(t)$.

Linearity

- Note, as a consequence, we can show that fo linear system an input which is zero for all time results in an output which is zero for all time.



Linearity: examples

1. $y(t) = t \cdot x(t)$. → **linear**
2. $y(t) = x^2(t)$. → **non-linear**
3. $y(t) = 2x(t) + 3$ → **non-linear**

Important properties

Comments on system properties

Study these properties at home

- Every memoryless system is causal
- The outuput of a linear system for a zero input is a zero output.
- If a system is time invariant, periodic inputs lead to periodic outputs.

Homeworks

- Study the properties of both systems:

$$y(t) = \begin{cases} 0: & t < 0 \\ x(t) + x(t-2): & t \geq 0 \end{cases}$$

$$y(t) = \begin{cases} 0: & x(t) < 0 \\ x(t) + x(t-2): & x(t) \geq 0 \end{cases}$$

Summary: what we saw so far

- what is a system
- properties of a system

Summary: what will see…

- We will focus on: LINEAR TIME INVARIANT (LTI) SYSTEMS
- LTI systems in time
- LTI systems in transformed domain (frequency domain etc.)

Questions?