Topic 2 - Part 2: SYSTEMS IN THE TIME DOMAIN

Linear systems and circuit applications

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Based also on Professor Óscar Barquero Perez, Andrés Martínez and José Luis Rojo's slides

What will see…

- We will focus on: LINEAR TIME INVARIANT (LTI) SYSTEMS
- LTI systems in time
- LTI systems in transformed domain (frequency domain etc.)
- a system is also called as *filter*

In this slides of Topic 2…

- **LTI systems in time !!!!**
- **How to deal *mathematically* with LTI systems in time.**
- **(recall, *in this course*, we are in *continuous time*…)**

How express an LTI system

- Mathematically, the output the LTI systems can be expressed in two ways:

1. Linear differential equations with constant coefficients and null initial conditions.
2. Convolution integral.

- These two ways are equivalent.

Linear ordinary differential equations (L-ODE)

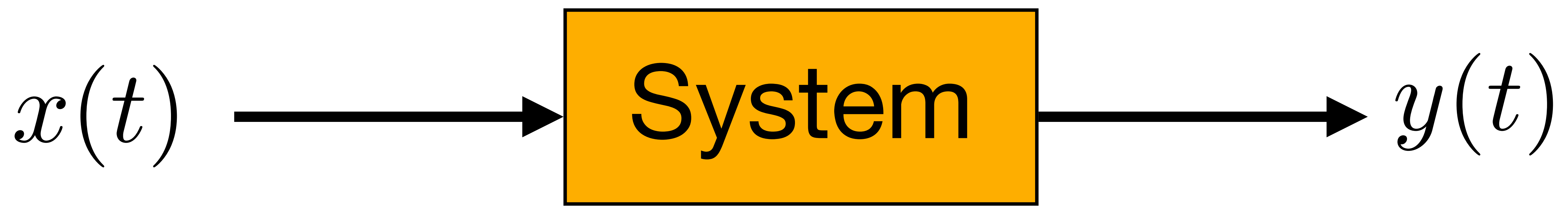
1. Linear differential equations with constant coefficients and null initial conditions:

$$\sum_{n=0}^{N} a_n \frac{d^n y(t)}{dt^n} = \sum_{m=0}^{M} b_m \frac{d^m x(t)}{dt^m}$$

(N) Initial conditions:

$$y(0) = \left.\frac{dy(t)}{dt}\right|_{t=0^-} = \ldots = \left.\frac{d^{N-1}y(t)}{dt^{N-1}}\right|_{t=0^-} = 0$$

- **x(t): input ——> y(t): output**



We are interested in this ***"forced" L-ODE with null initial conditions***

Linear ordinary differential equations (L-ODE)

1. **Linear differential equations with constant coefficients and null initial conditions, examples:**

$$\frac{d^2y(t)}{dt^2} - 2\frac{dy(t)}{dt} = x(t) \longrightarrow \begin{array}{l} N = 2; \quad a_2 = 1; \quad a_1 = -2 \quad \text{the rest of } a_n \text{ are zero} \\ M = 0; \quad b_0 = 1; \quad \text{the rest of } b_m \text{ are zero} \end{array}$$

$$\frac{dy(t)}{dt} - 4.5y(t) = x(t) + j\frac{dx(t)}{dt} + 6\frac{d^2x(t)}{dt^2} \longrightarrow \begin{array}{l} N = 1; \quad a_1 = 1; \quad a_0 = -4.5 \quad \text{the rest of } a_n \text{ are zero} \\ M = 2; \quad b_0 = 1; b_1 = j; b_2 = 6 \quad \text{the rest of } b_m \text{ are zero} \end{array}$$

Brief overview of the solutions of L-ODE

- **First of all, define the *homogeneous L-ODE*:**

$$\sum_{n=0}^{N} a_n \frac{d^n y(t)}{dt^n} = 0$$

- **with general *non-null* initial conditions:**

$$y(0) \neq 0 \quad \left.\frac{dy(t)}{dt}\right|_{t=0^-} \neq 0 \qquad \left.\frac{d^{M-1} y(t)}{dt^{N-1}}\right|_{t=0^-} \neq 0$$

Brief overview of the solutions of L-ODE

- The solution of an *homogeneous L-ODE* ***with NULL initial conditions is***:

$$y(t) = 0, \quad \forall t$$

- **But** with non-null initial conditions, the solution y(t) of an homogeneous L-ODE is non-zero (generally) and it is called ***transient solution,*** denoted here as:

$$y_o(t)$$

There are courses just devoted to study this solution…

Four cases…

1. homogeneous L-ODE with NULL initial conditions
2. homogeneous L-ODE with NON-NULL initial conditions
3. forced L-ODE (with input) with NULL initial conditions
4. forced L-ODE (with input) with NON-NULL initial conditions

The more general case is the last one

But we are interested in the third one

Forced L-ODE with NON-NULL initial conditions

- **General solution:**

$$y(t) = y_o(t) + y_f(t)$$

Solution of the homogenous equation with NON-NULL initial conditions

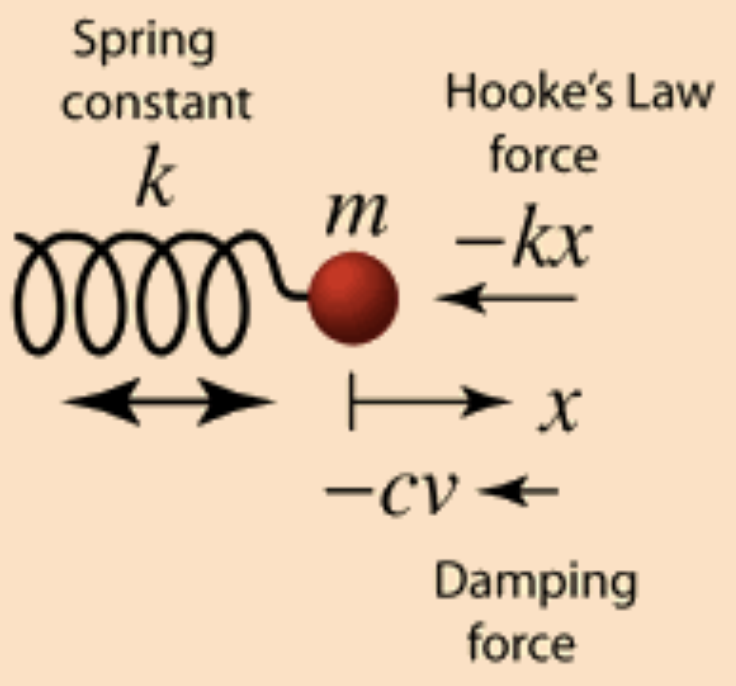
Solution of the forced equation with NULL initial conditions

Forced L-ODE with NON-NULL initial conditions

Driven Oscillator

If a damped oscillator is driven by an external force, the solution to the motion equation has two parts, a transient part and a steady-state part, which must be used together to fit the physical boundary conditions of the problem.

The motion equation is of the form



Newton's 2nd Law terms — Sinusoidal driving force

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=F_0\cos(\omega t+\varphi_d)$$

and has a general solution

$$x(t)=x_{transient}+x_{steady\ \ state}$$

In the underdamped case this solution takes the form

Transient solution — Steady-state solution

$$x(t)=A_h e^{-\gamma t}\sin(\omega' t+\varphi_h)+A\cos(\omega t-\varphi)$$

Determined by initial position and velocity — Determined by driving force

The initial behavior of a damped, driven oscillator can be quite complex. The parameters in the above solution depend upon the initial conditions and the nature of the driving force, but deriving the detailed form is an involved algebra problem. The form of the parameters is shown below.

different notation

$$y(t)=y_o(t)+y_f(t)$$

Forced L-ODE with NULL initial conditions

- **In this case:**

$$y(t) = 0 + y_f(t)$$

Solution of the homogenous equation with NULL initial conditions

Solution of the forced equation with NULL initial conditions

- ***In this course, we focus on this scenario.***

Forced L-ODE with NULL initial conditions

- **In this case:**

$$y(t) = y_f(t)$$

→ **Solution of the forced equation with NULL initial conditions**

- ***In this course, we focus on this solution.***
- all the books/notes use the notation y(t) but is actually y_f(t)

Convolution integral

- **Solution of the forced equation with NULL initial conditions can be as:**

$$y(t) = y_f(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$$

- *but what is the function/signal h(t) ?*

Convolution integral

- **Convolution:**

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$$

- *h(t) is called* **IMPULSE RESPONSE (respuesta al impulso)**
- **impulse = delta function**

Convolution integral

- **Other notation:**

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$$

- *h(t) is called* **IMPULSE RESPONSE (respuesta al impulso)**
- **impulse = delta function**

Impulse Response

- Reason of the name: **h(t) is the response of the system when the input is a delta (an impulse).**
- h(t) is the output, i.e., y(t)=h(t), of the system when x(t)=delta(t):

$$x(t) = \delta(t) \Longrightarrow y(t) = h(t)$$

Impulse Response

- h(t) is the output, i.e., y(t)=h(t), of the system when x(t)=delta(t):

$$x(t) = \delta(t) \Longrightarrow y(t) = h(t)$$

- indeed, for the delta's properties:

$$y(t) = \int_{-\infty}^{+\infty} \delta(\tau) h(t-\tau) d\tau = h(t)$$

Impulse Response

- **Then, h(t) summaries an LTI system (and its properties).**

- Then, h(t) “summaries”/is equivalent a forced L-ODE with null initial conditions.

Summary: what we saw in these slides

- An LTI system in continuous time can be expressed (mathematically):
- Forced L-ODE *with constant coefficients* and *with NULL initial conditions*
- Convolution integral where h(t) is the impulse response

Next….

- properties of the convolution

Questions?