Topic 2 - Part 5: Convolution - more examples and properties

Systems in time domain

Linear systems and circuit applications

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Based also on Professor Óscar Barquero Perez, Andrés Martínez and José Luis Rojo's slides

In this slides…

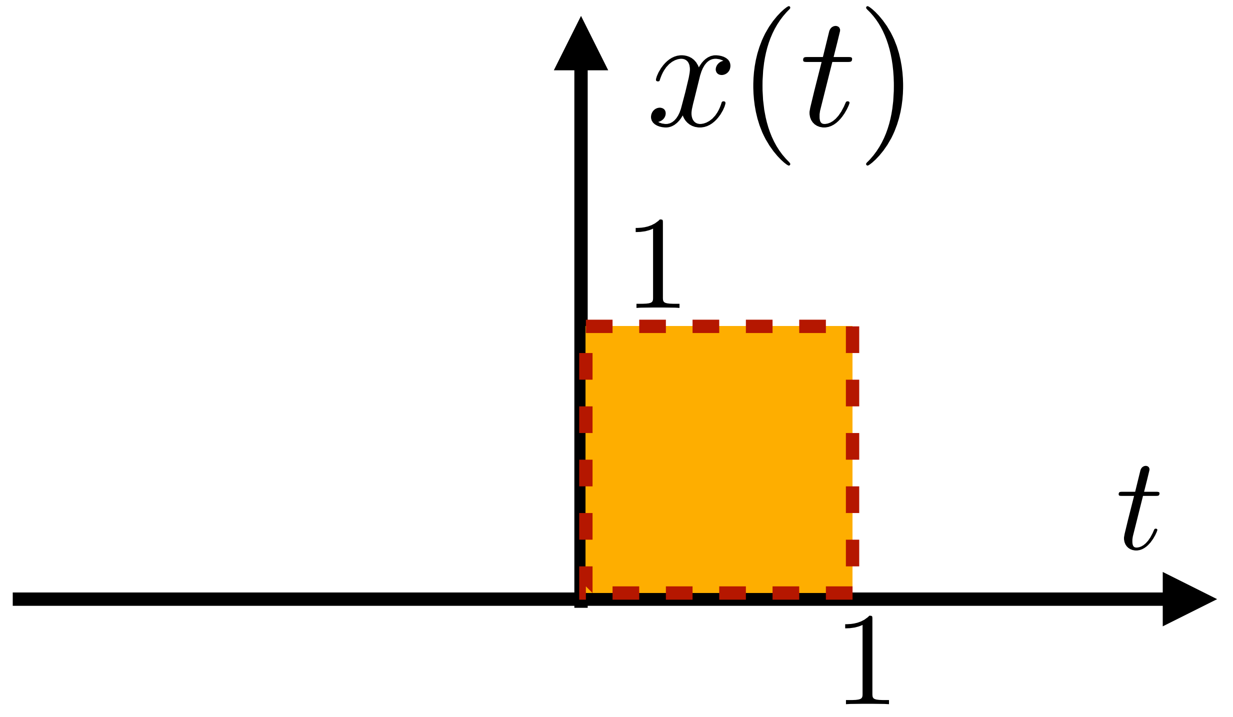
- **Other examples - problems - exercises**

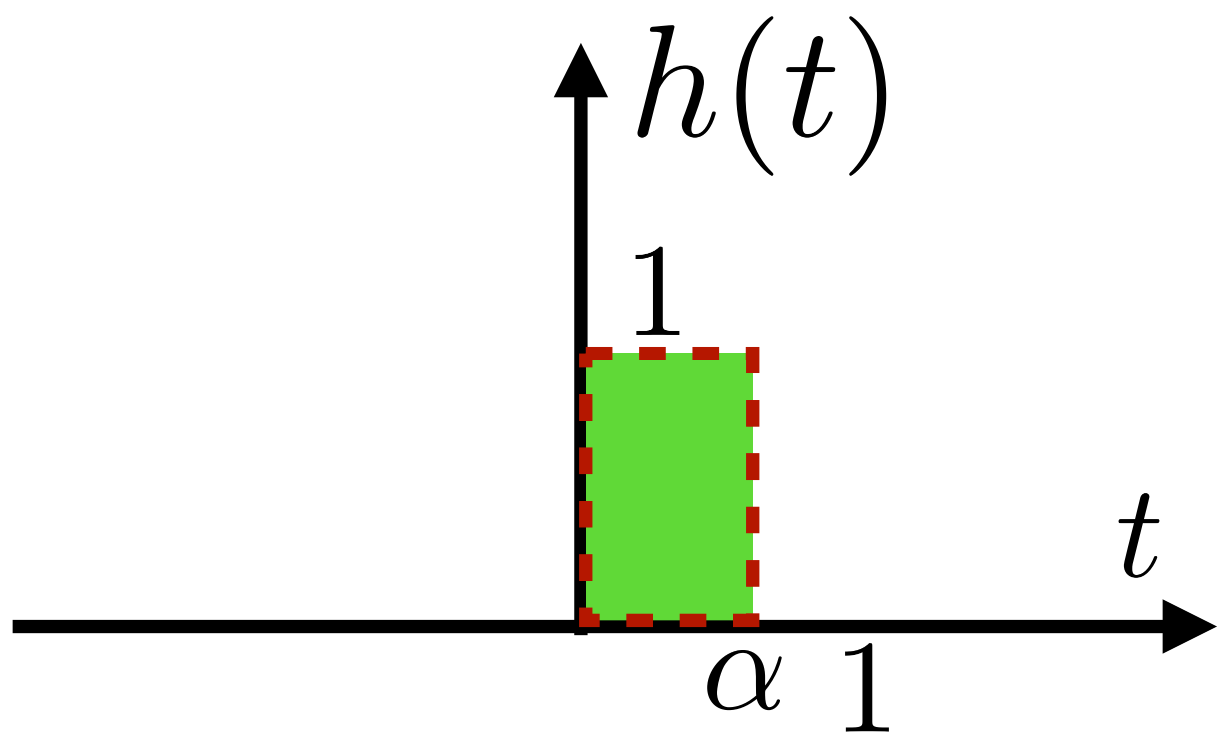
Recall - Example 6 (see previous slides)

$x(t) = 1$ if $t \in [0, 1]$, otherwise $x(t) = 0$

$h(t) = 1$ if $t \in [0, \alpha]$, otherwise $h(t) = 0$ with $\alpha < 1$

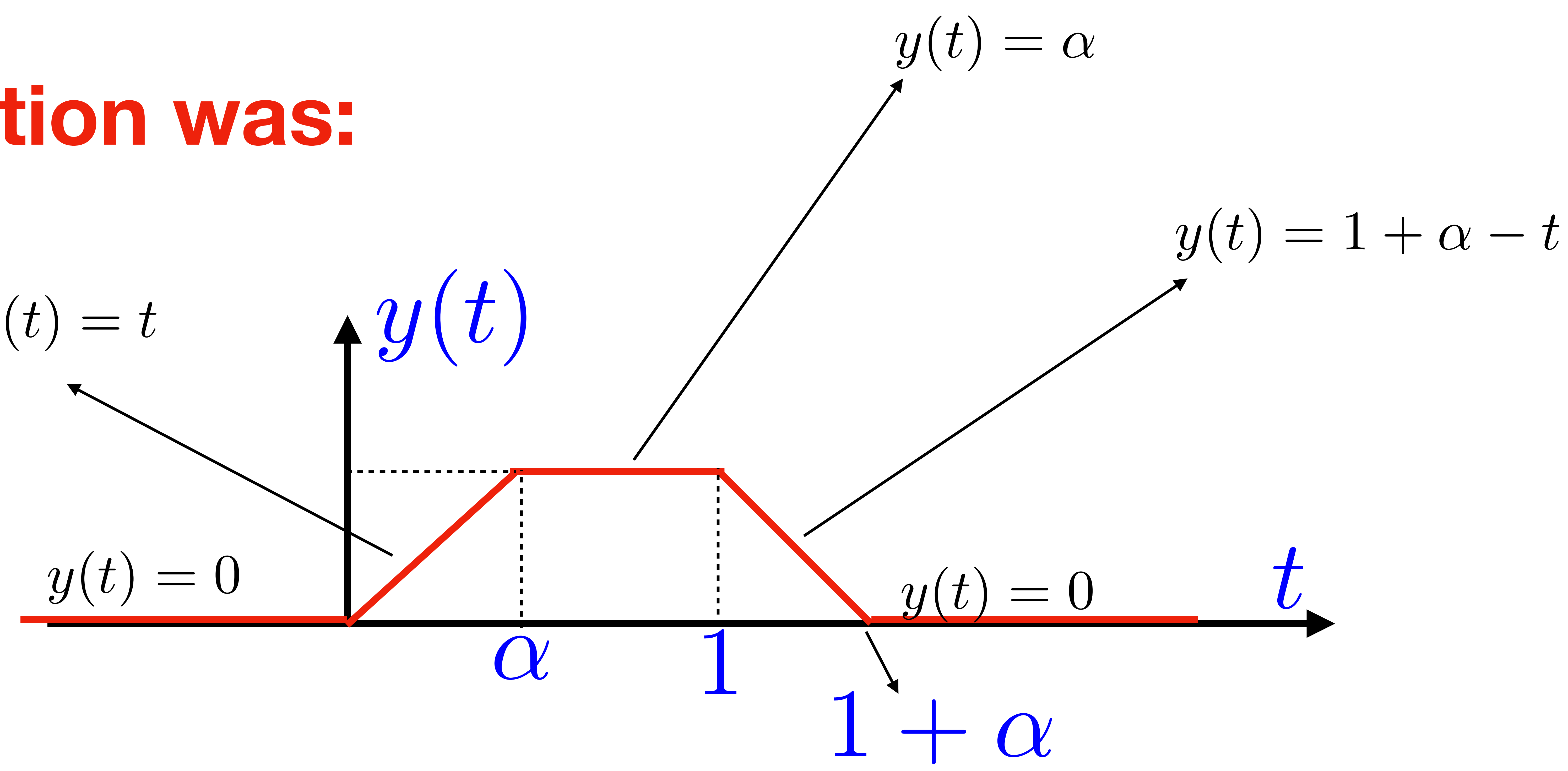
$$y(t) = x(t) * h(t) = ?$$



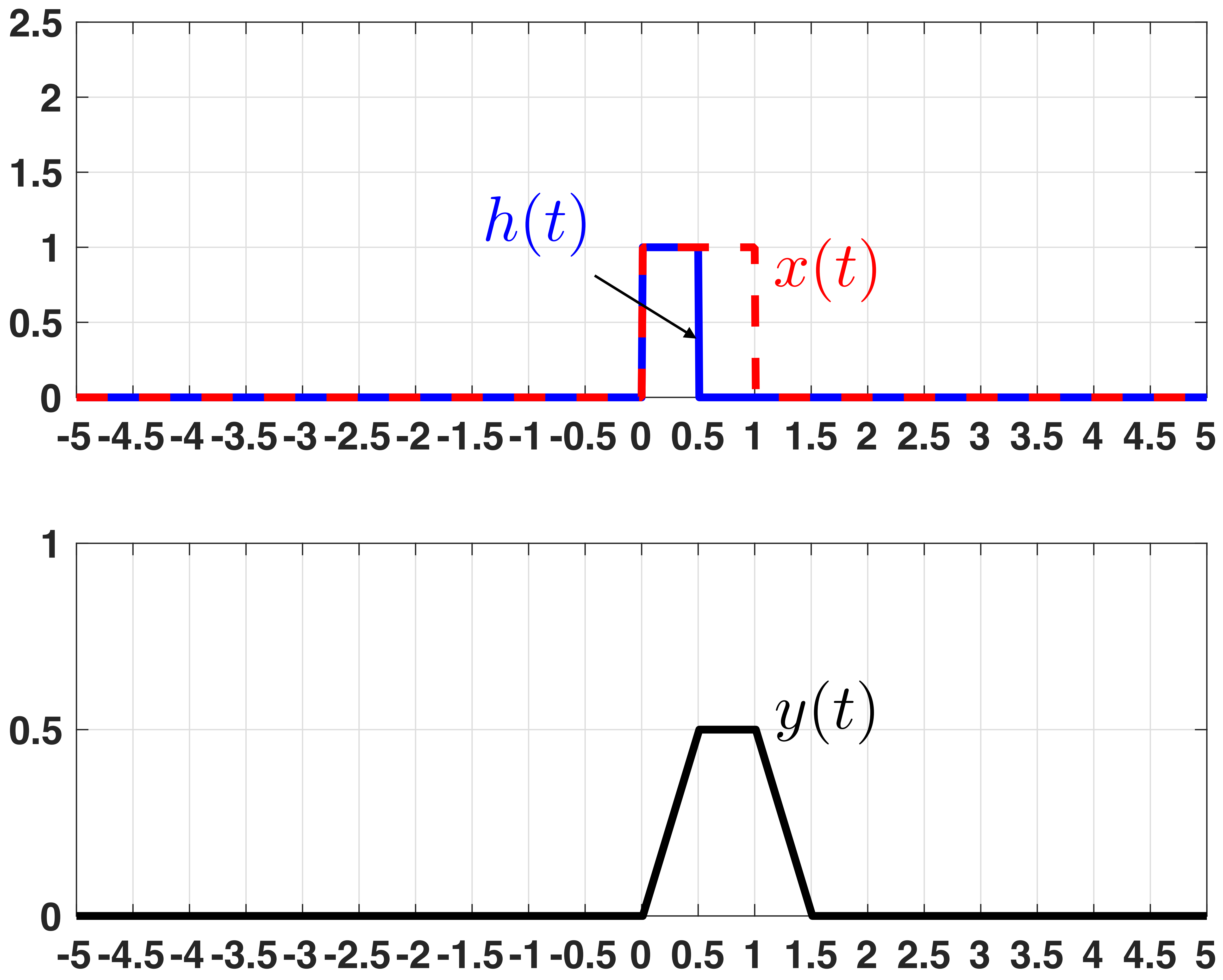


Recall - Example 6 (see previous slides)

- **The final solution was:**



Example 6 - just other view



- Solution

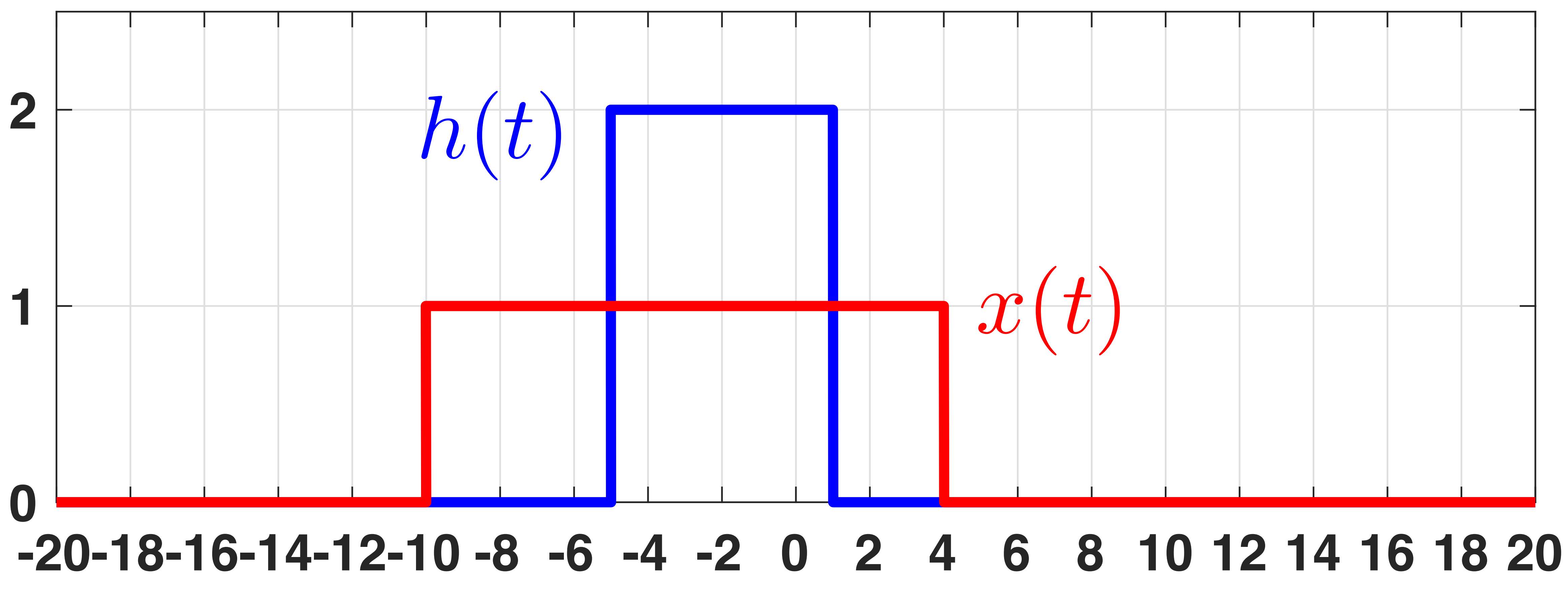
Example 7

$$x(t) = 1 \text{ if } t \in [-10, 4], \text{ otherwise } x(t) = 0$$

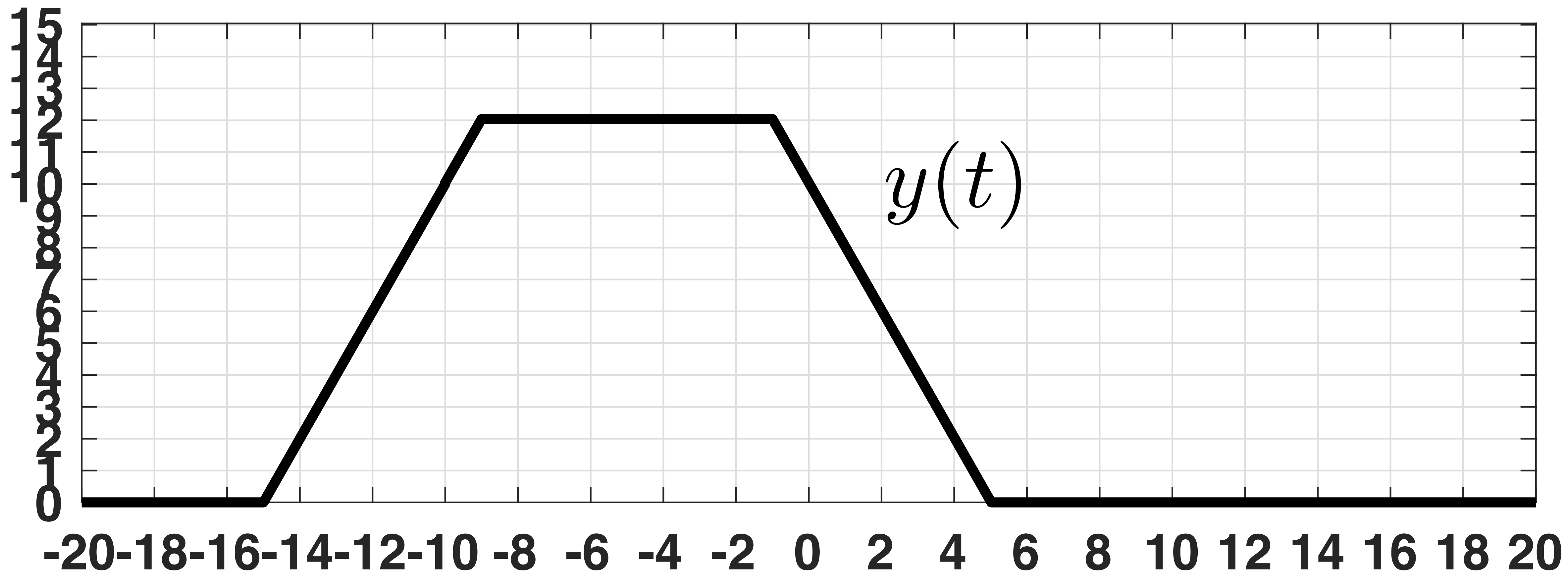
$$h(t) = 2 \text{ if } t \in [-5, 1], \text{ otherwise } h(t) = 0$$

$$y(t) = x(t) * h(t) = ?$$

Example 7



- **Solution**

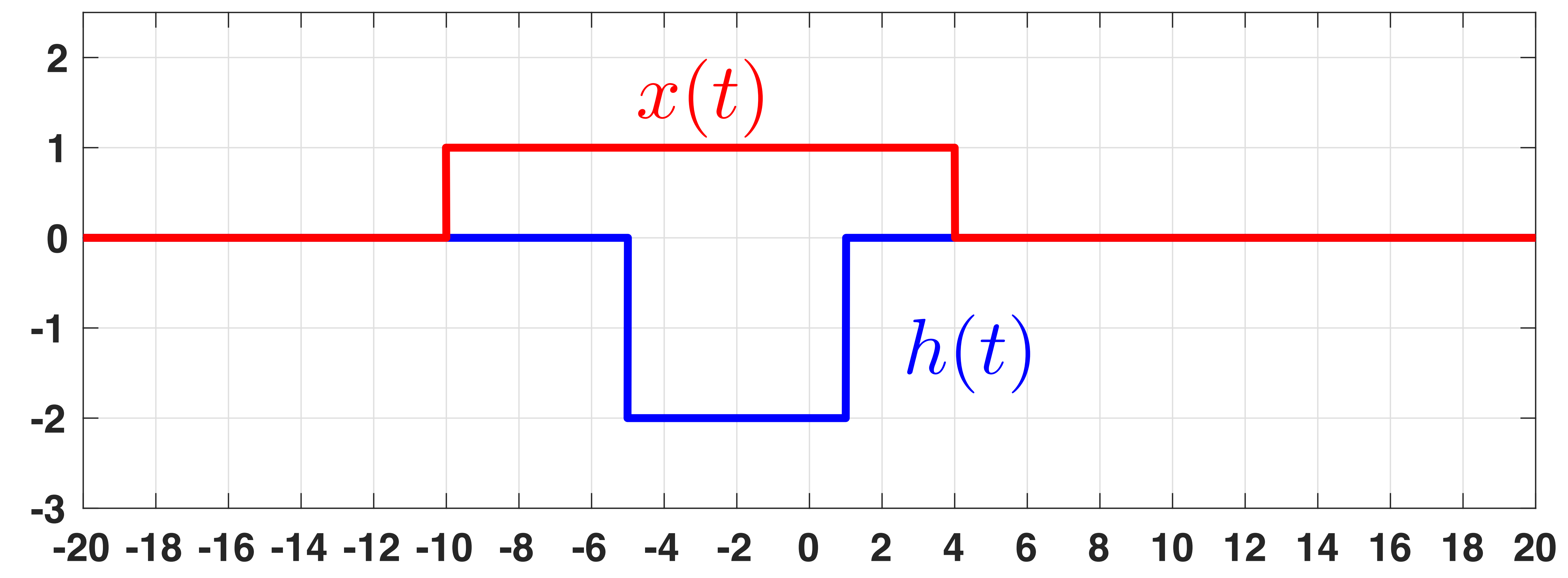


Example 8

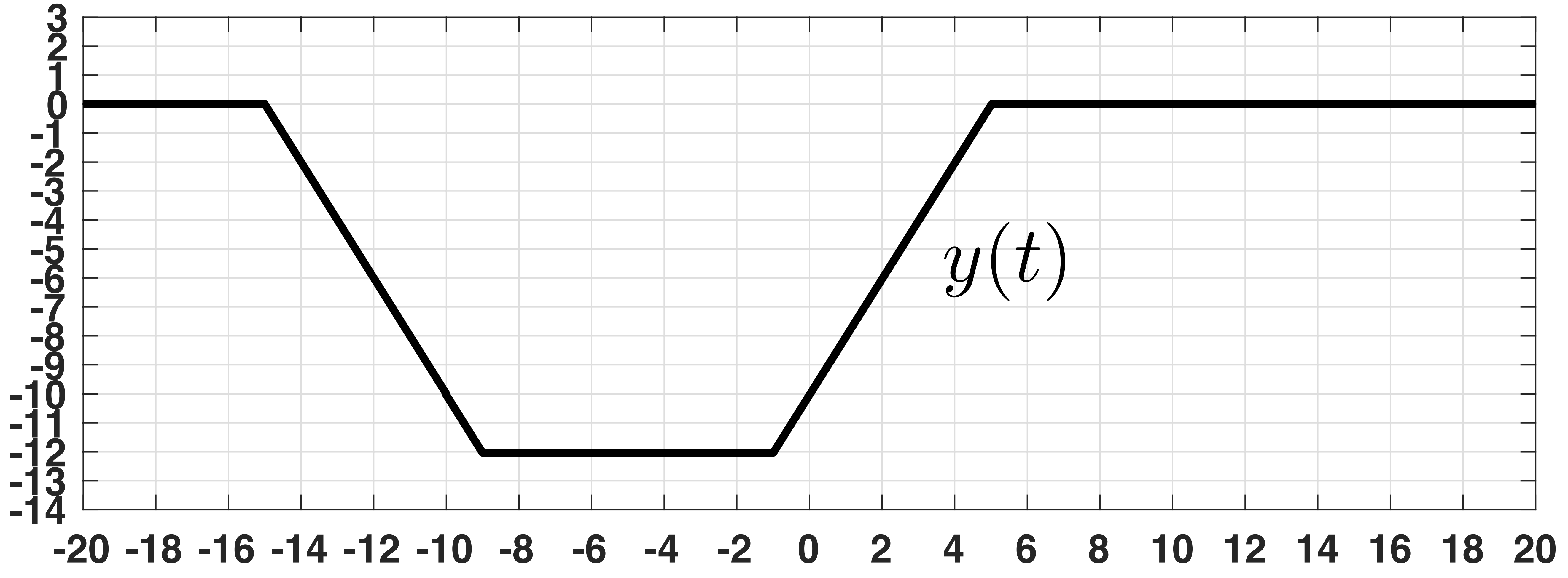
$$x(t) = 1 \text{ if } t \in [-10, 4], \text{ otherwise } x(t) = 0$$
$$h(t) = -2 \text{ if } t \in [-5, 1], \text{ otherwise } h(t) = 0$$

$$y(t) = x(t) * h(t) = ?$$

Example 8



- **Solution**



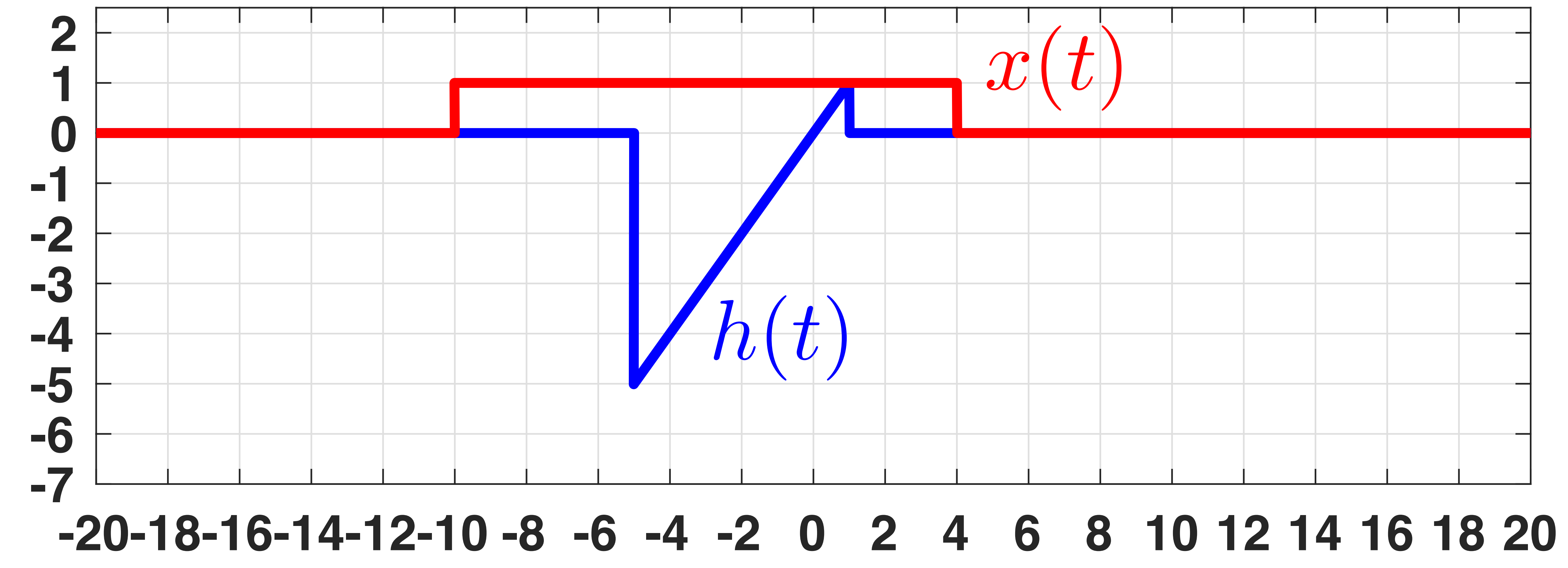
Example 9

$$x(t) = 1 \text{ if } t \in [-10, 4], \text{ otherwise } x(t) = 0$$

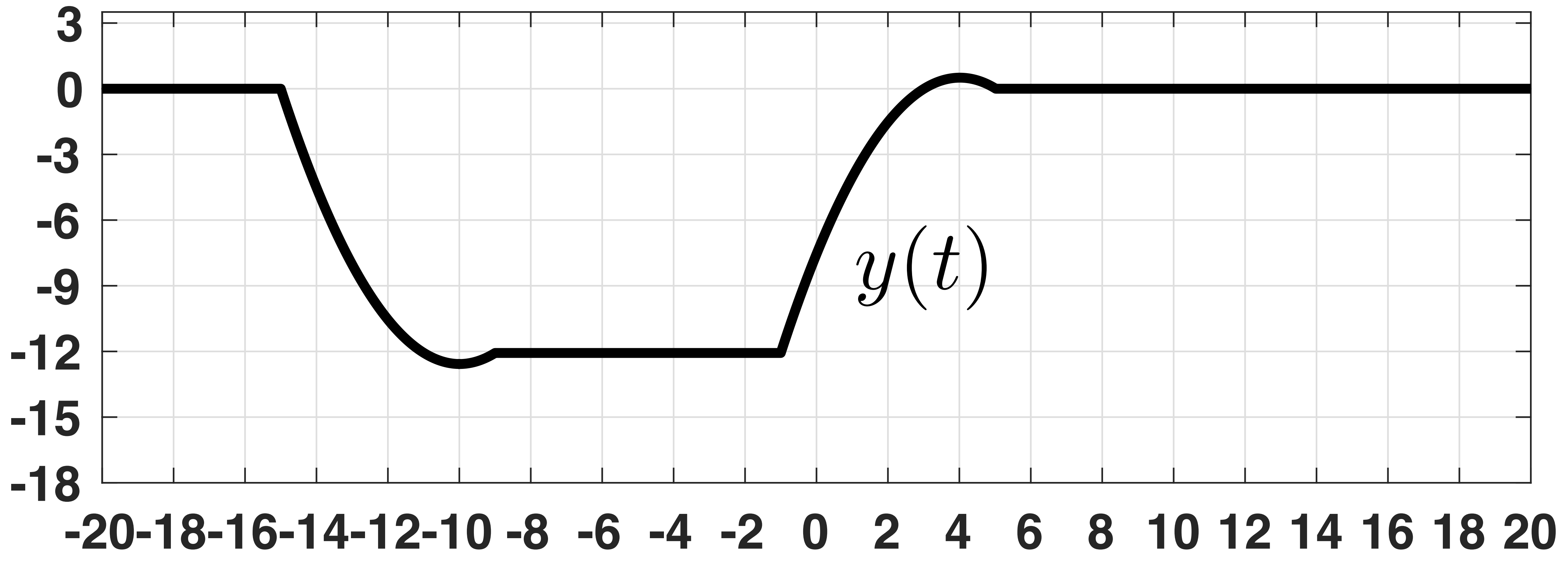
$$h(t) = t \text{ if } t \in [-5, 1], \text{ otherwise } h(t) = 0$$

$$y(t) = x(t) * h(t) = ?$$

Example 9



- **Solution**



Example 10

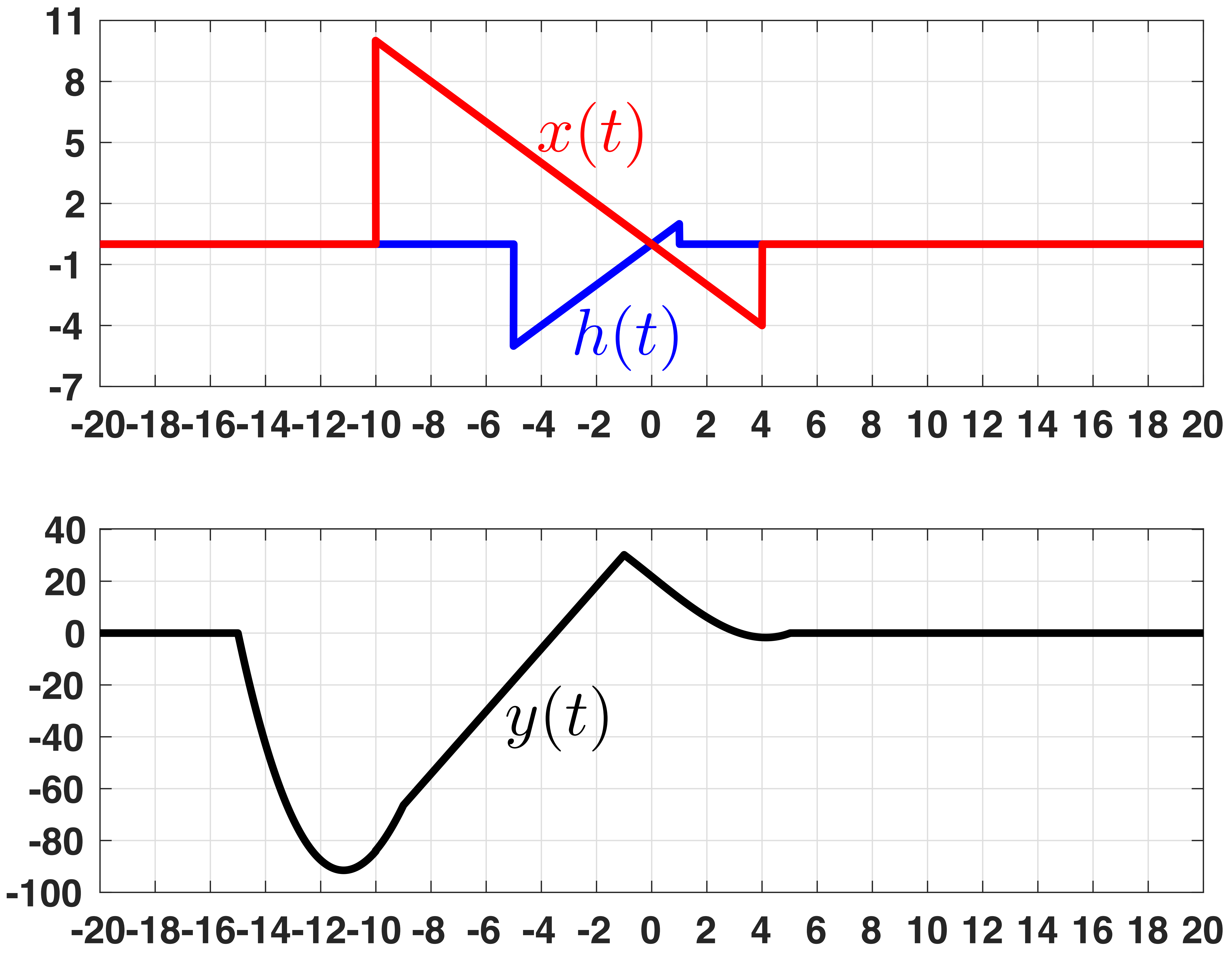
$$x(t) = -t \text{ if } t \in [-10, 4], \text{ otherwise } x(t) = 0$$

$$h(t) = t \text{ if } t \in [-5, 1], \text{ otherwise } h(t) = 0$$

$$y(t) = x(t) * h(t) = ?$$

Example 10

- **Solution**



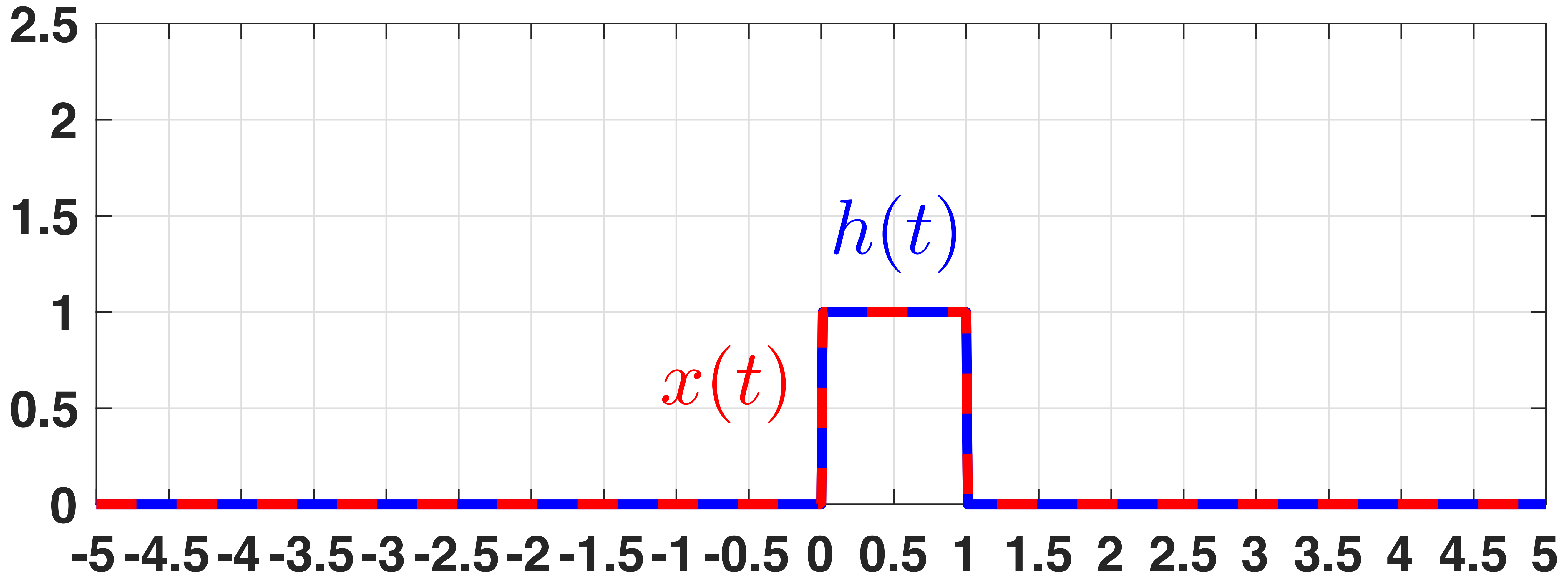
Example 11

$$x(t) = 1 \text{ if } t \in [0, 1], \text{ otherwise } x(t) = 0$$

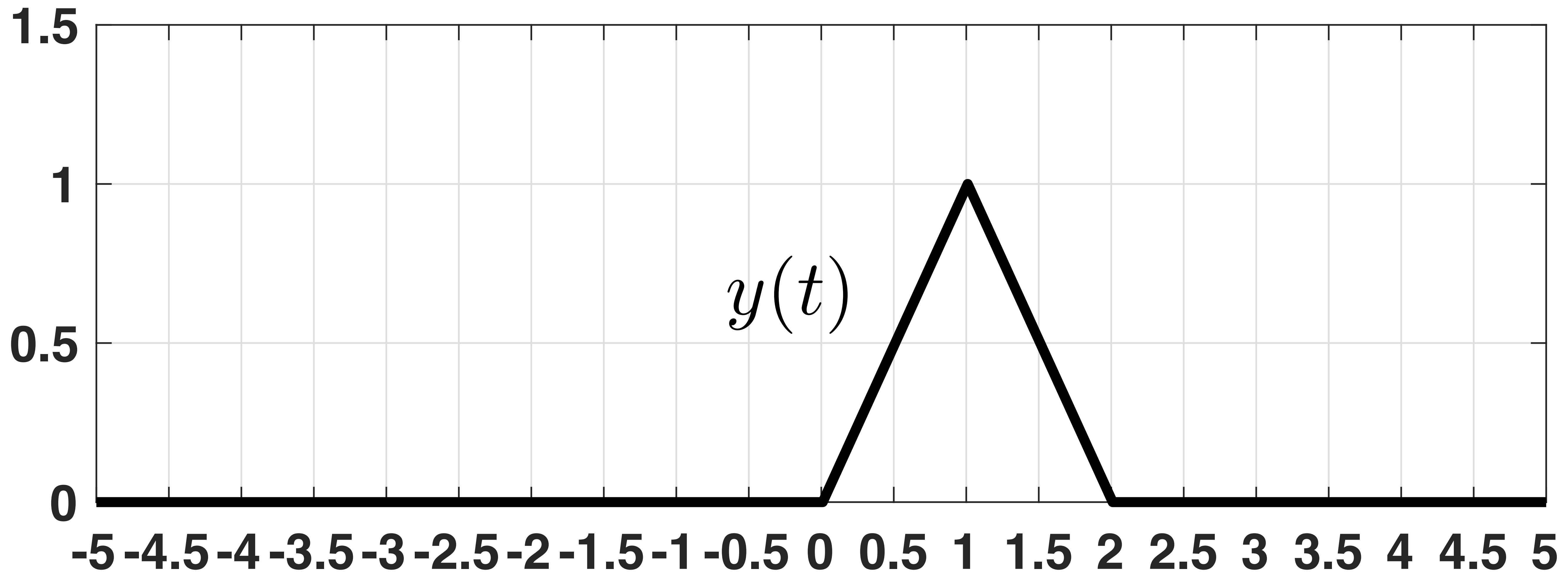
$$h(t) = 1 \text{ if } t \in [0, 1], \text{ otherwise } h(t) = 0$$

$$y(t) = x(t) * h(t) = ?$$

Example 11



- **Solution**



Example 12

$$x(t)$$

$$h(t) = u(t)$$

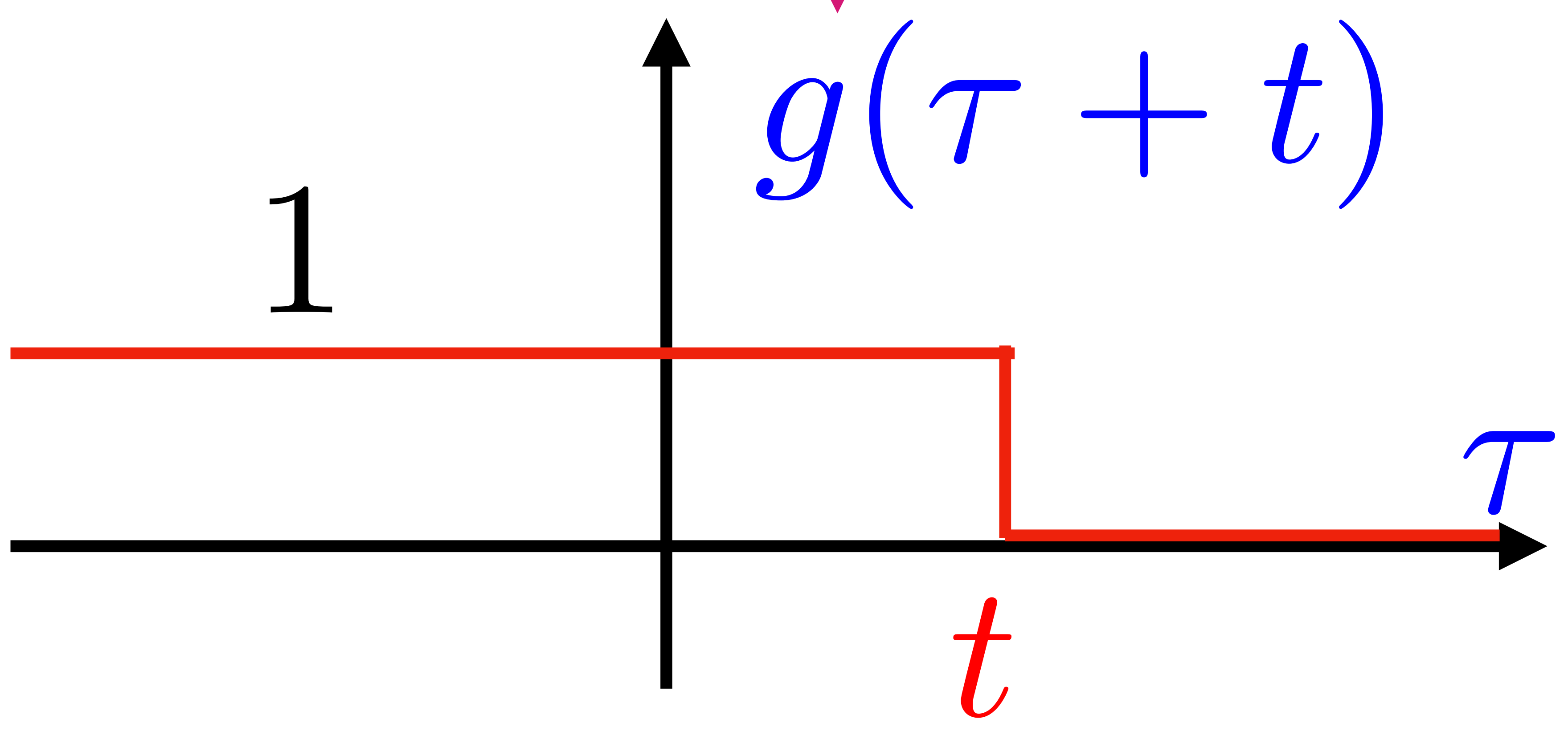
• **step function**

$$y(t) = x(t) * h(t) = ?$$

Example 12

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)u(t-\tau)d\tau$$

$$g(\tau) = u(-\tau)$$



Example 12

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)u(t-\tau)d\tau$$

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

- **An integrator**

More examples

- More complicated problems at:

https://www.youtube.com/watch?v=znkTOn0SfYU&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=126

https://www.youtube.com/watch?v=rzV1iWmqtX0&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=124

Important properties

- Two signals - x(t) and h(t) - with **FINITE LENGTHS:**

$$x(t) \text{ non-zero in } t \in [t_1, t_2],$$

$$h(t) \text{ non-zero in } t \in [t_3, t_4],$$

$$\Downarrow$$

$$y(t) \text{ non-zero in } t \in [t_1 + t_3, t_2 + t_4],$$

$$t_1 + t_3 \leq t \leq t_2 + t_4$$

Important properties

- Property of the areas (integrals in the domain):

$$y(t) = x(t) * h(t)$$

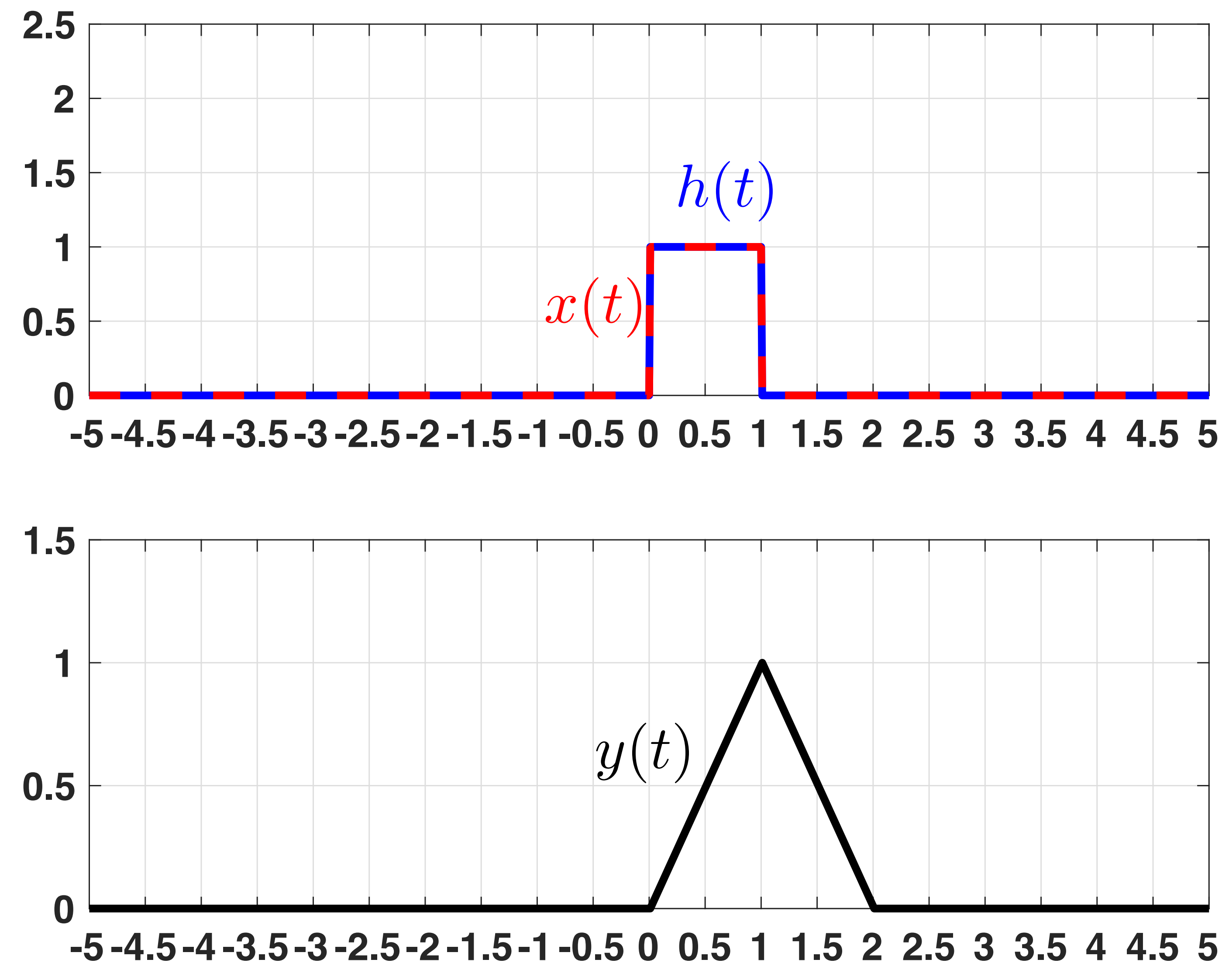
$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$A_Y \qquad A_X \qquad A_H$$

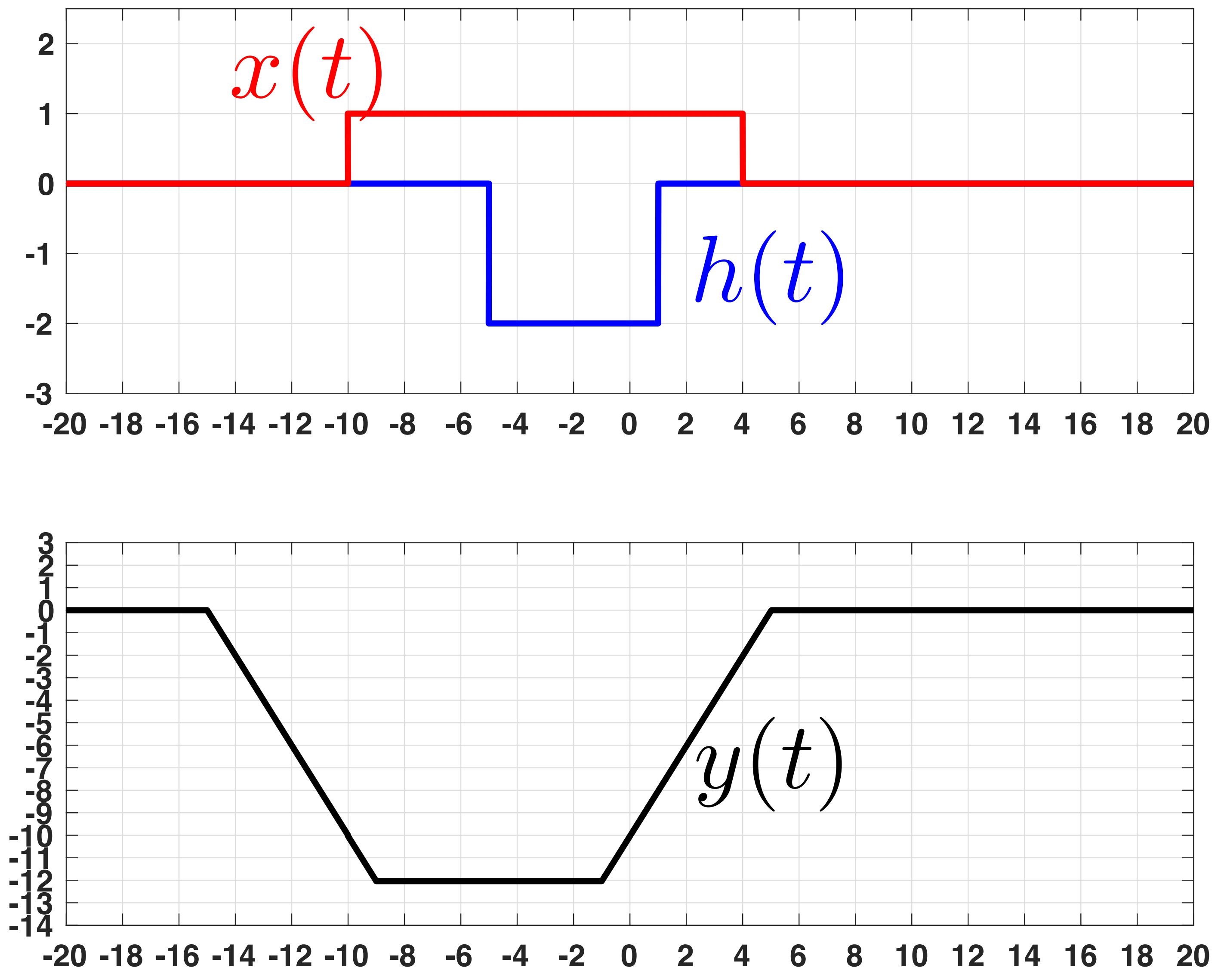
$$A_Y = A_X A_H$$

Important properties

- Consider two signals - x(t) and h(t) - with **FINITE LENGTHS**, for instance, one with length Tx and the other with length Th.
- **Then signal obtained as convolution, y(t), the LENGTH is Ty=Tx+Th.**



$$T_X = 1$$
$$T_H = 1$$
$$T_Y = 2$$



$$T_X = 14$$
$$T_H = 6$$
$$T_Y = 20$$

Important properties

- related links:

https://www.youtube.com/watch?v=80uvr_tdSI8&list=PLBlnK6fEyqRhG6s3jYIU48CqsT5cyiDTO&index=119

Questions?