continuous signal (in time)
non-periodic transform (in frequency)

periodic signals (in time)
discrete sequence (in frequency)

Fourier Series of a continuous periodic signal

Fourier Series (FS)

period

For $x(t)$ of duration T, set $\omega_0 = \frac{2\pi}{T}$.

periodic $x(t)$: $0 \leq t \leq T$
$X[k]$: $k = \ldots, -2, -1, 0, 1, 2, \ldots$

$$a_k = X[k] = \frac{1}{T}\int_{t=0}^{T} x(t)\, e^{-jk\omega_0 t} dt$$

$$x(t) = \sum_{k=-\infty}^{\infty} X[k]\, e^{jk\omega_0 t}$$

non-periodic signals (in time)
continuous function (in frequency)

Fourier Transform of a continuous signal

Fourier Transform (FT)

$x(t)$: $-\infty < t < \infty$
$X(\omega)$: $-\infty < \omega < \infty$

$$X(\omega) = \int_{t=-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} X(\omega)\, e^{j\omega t} d\omega$$

dicrete signal (in time)
periodic transform (in frequency)

Fourier Series of a discrete periodic signal

Discrete Fourier Transform (DFT)

period

For $x[n]$ of length N, set $\omega_0 = \frac{2\pi}{N}$.

periodic $x[n]$: $n = 0, 1, \ldots, N-1$
periodic $X[k]$: $k = 0, 1, \ldots, N-1$

$$a_k = X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-jk\omega_0 n}$$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{jk\omega_0 n}$$

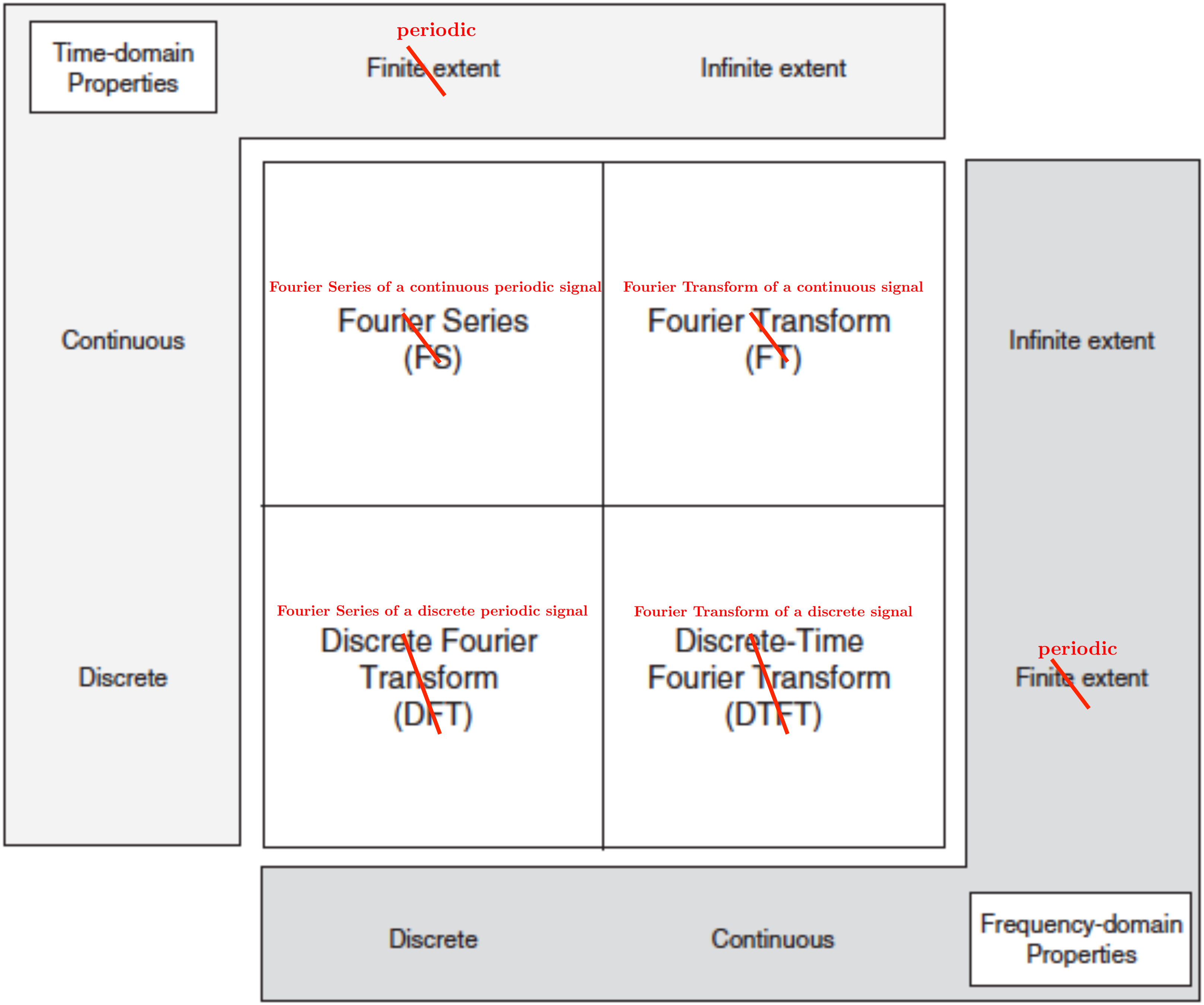
Fourier Transform of a discrete signal

Discrete-Time Fourier Transform (DTFT)

$x[n]$: $n = \ldots, -2, -1, 0, 1, 2, \ldots$
periodic $X(\omega)$: $-\pi \leq \omega \leq \pi$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{\omega=-\pi}^{\pi} X(\omega)\, e^{j\omega n} d\omega$$


Time-domain Properties
periodic
Finite extent
Infinite extent
Continuous
Discrete
Fourier Series of a continuous periodic signal
Fourier Series (FS)
Fourier Transform of a continuous signal
Fourier Transform (FT)
Fourier Series of a discrete periodic signal
Discrete Fourier Transform (DFT)
Fourier Transform of a discrete signal
Discrete-Time Fourier Transform (DTFT)
Infinite extent
periodic
Finite extent
Discrete
Continuous
Frequency-domain Properties