OVERVIEW

Linear systems and circuit applications
Discrete Time Systems, Señales y Sistemas

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Transformations for signal in continuous time

For Periodic signals	For non-periodic signals	
Fourier Series (FS)	Stand. Fourier Transform (FT)	Laplace Transform (LT)

also for some Signals with Infinite Energy

Generalized Fourier Transform (GFT)

Mathematically, it is not completely valid… or we need other definition of Fourier Transformation….

Transformations for signal in discrete time

For Periodic signals | **For non-periodic signals**

Fourier Series (FS)

Stand. Fourier Transform (FT)

Zeta Transform (ZT)

also for some Signals with Infinite Energy

Generalized Fourier Transform (GFT)

Mathematically, it is not completely valid… or we need other definition of Fourier Transformation….

Fourier series….

The existence of the Fourier series is the "proof" that the Standard Fourier transform (WITH OUR MATH) cannot be used for periodic signals…

The LAPLACE AND ZETA TRANSFORMS generalize the Standard Fourier transform…

Transformations for signal in discrete time

MOREOVER: (practical tools…)

- **Discrete Fourier Transform (DFT)**
- **Fast Fourier Transform (FFT) ==> Fast version of DFT**

DFT “mathematically” very similar to (almost the same) Fourier Series for discrete time

	Periódica en el tiempo	No periódica en el tiempo	
Continua en el tiempo $\omega_0 = \frac{2\pi}{T}$	**CTFS** $a_k = X[k] = \frac{1}{T}\int_{<T>} x(t) e^{-jk\omega_0 t} dt$ $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	**CTFT** $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$ $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$	No periódica en frecuencia
Discreta en el tiempo $\Omega_0 = \frac{2\pi}{N}$	**DTFS** $x[n] = \sum_{k=<N>} a_k e^{jk\Omega_0 n}$ $a_k = X[k] = \frac{1}{N}\sum_{n=<N>} x[n] e^{-jk\Omega_0 n}$	**DTFT** $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega) e^{j\Omega n} d\Omega$ $X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$	Periódica en frecuencia
	Discreta en frecuencia	Continua en frecuencia	

WE WILL SEE:

(in this course)

FOURIER SERIES

STANDARD FOURIER TRANSFORM

Generalized Fourier Trasform (GFT)
- for a periodic signal in CT

For periodic signals

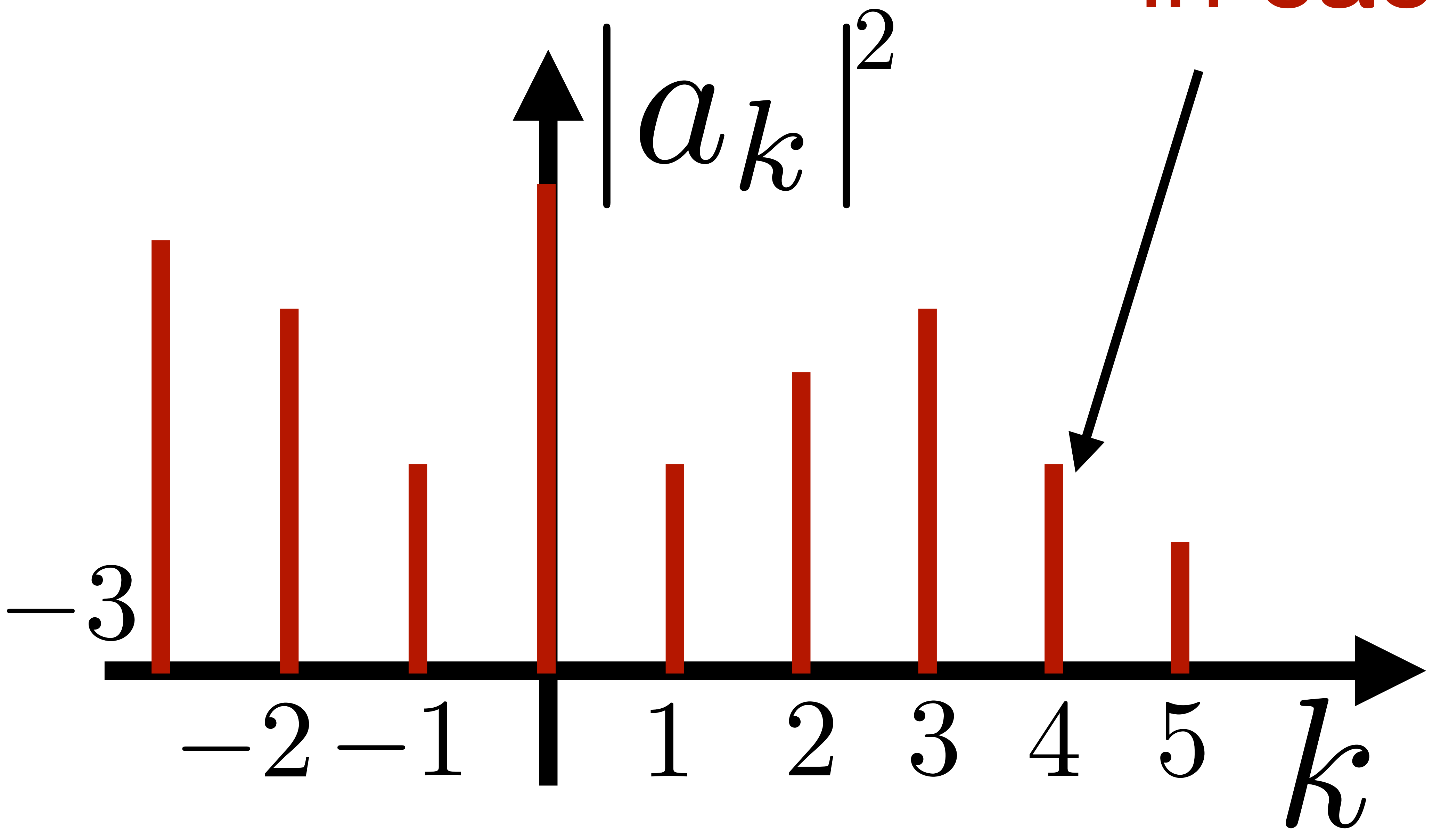
$$x(t) = x(t + T_0) \overbrace{- - - - - >}^{FS} a_k$$

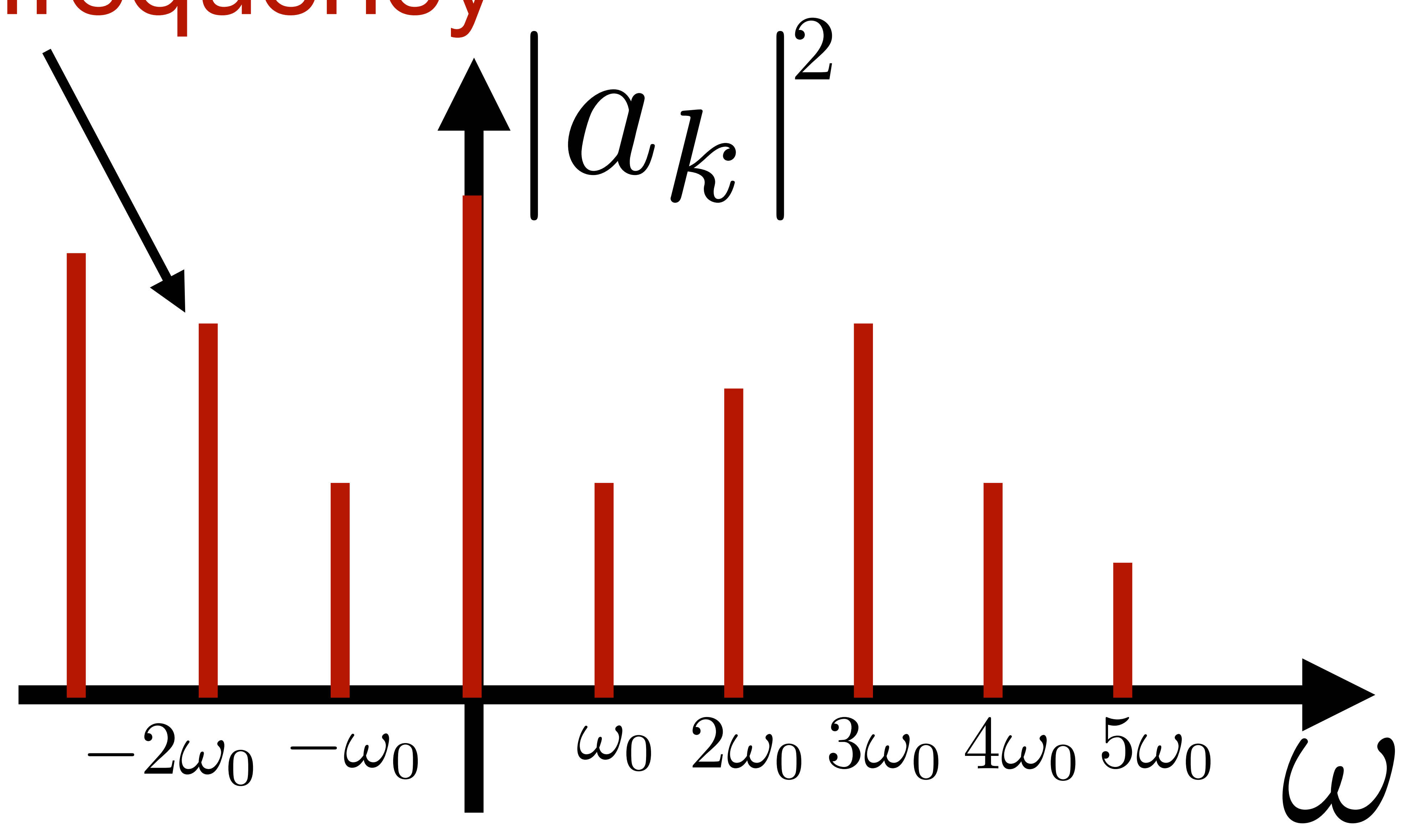
$$X_G(\omega) = 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$$

Recall something of the FS…

- The coefficients a_k's are complex numbers

- (square root of) ENERGY in each frequency

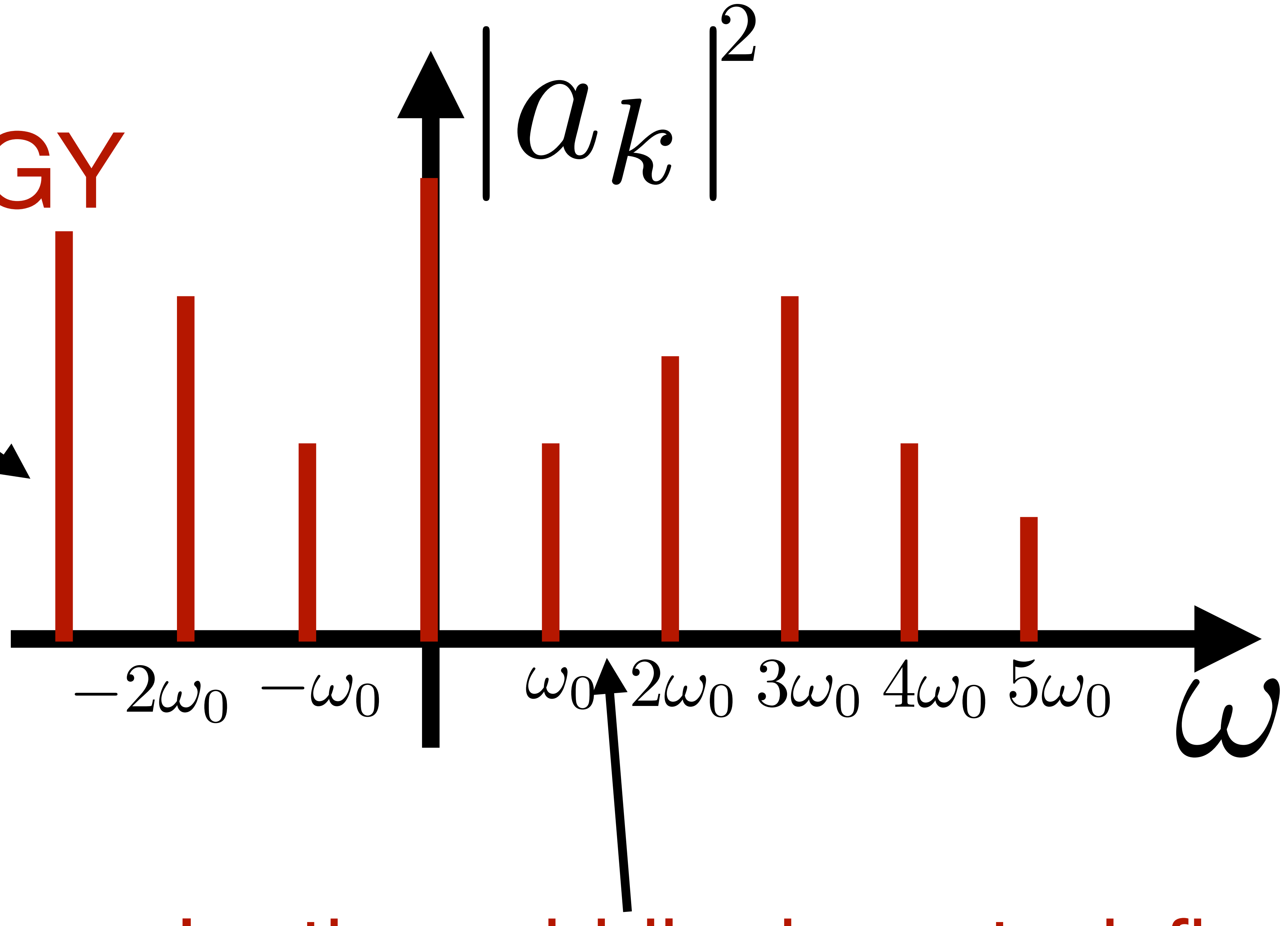




Recall something of the FS…

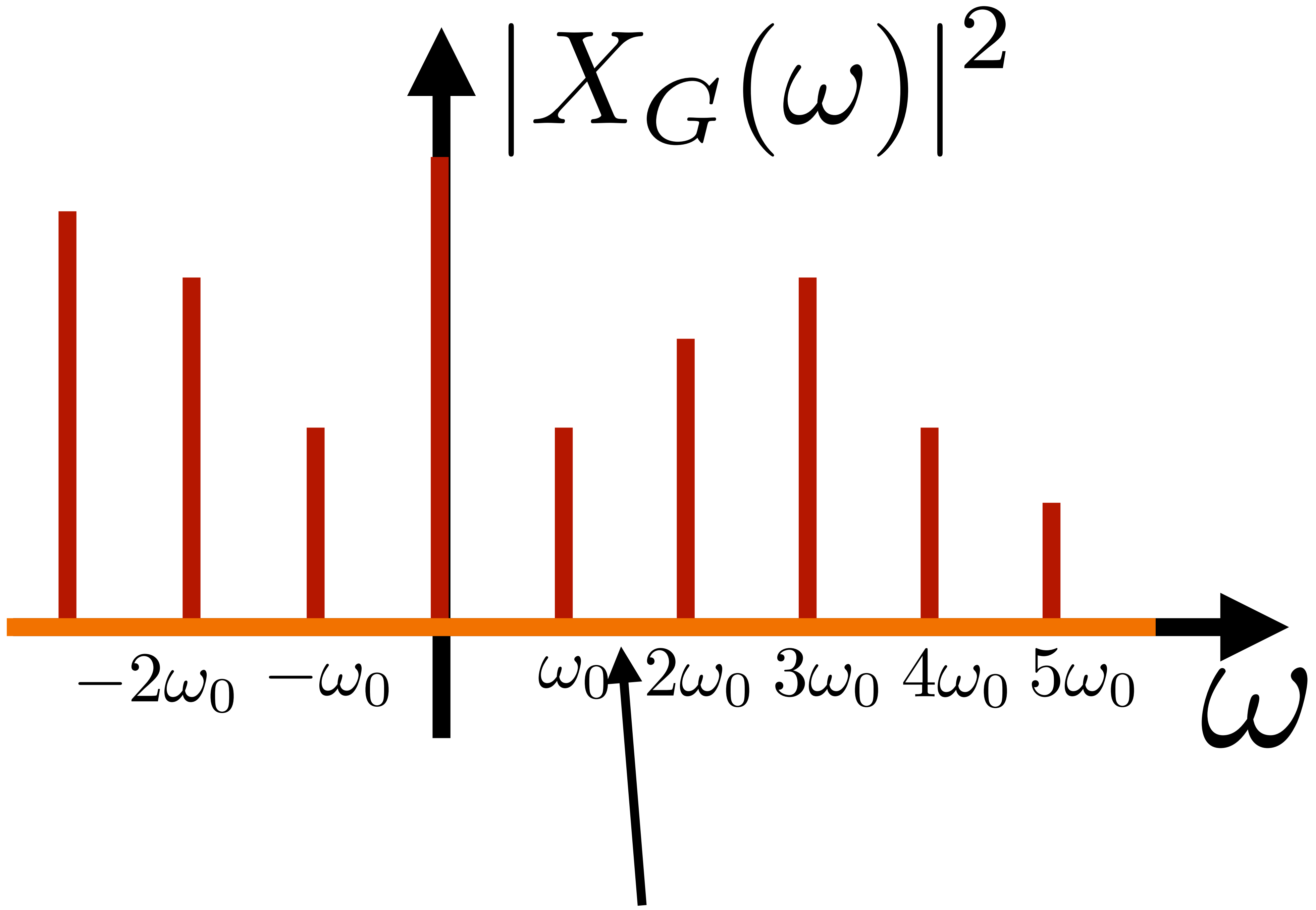
- Important observation:

- (square root of) ENERGY in each frequency



- In the middle is not defined… these frequencies are not contained

Recall something of the FS…



- If we consider a function that is equal to a_k at each kw_0 and “0” otherwise (there), then “almost nothing changes”…

Generalized Fourier Trasform (GFT) for a periodic signal in CT

For periodic signals

$$X_G(\omega) = 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$$

- It contains only the information of the FS
- we need to do the FS, to have the GFT

Generalized Fourier Trasform (GFT)
for a periodic signal in Discrete Time

For periodic signals

$$X_G(\Omega) = 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\Omega - k\Omega_0\right)$$

$$\Omega_0 = \frac{2\pi}{N}$$

- It contains only the information of the FS
- we need to do the FS, to have the GFT

Questions?