SOLVED problems - Convolution Sum - part 2

Discrete Time Systems (DTS)

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Example 5

Given:

$$x[0] = 2, \qquad x[n] = 0 \quad \text{otherwise}$$
$$h[-3] = -1, \quad h[n] = 0 \quad \text{otherwise}$$

*** Obtain the output of the LTI system y[n], with impulse response h[n]**
Namely, y[n]=?

*** Write the corresponding *linear difference equation* of this system (with constant coefficients and null initial conditions)**

Example 5

Then, in this example:

- start of x[n] at 0, start of h[n] at -3
- end of x[n] at 0, finish of h[n] at -3
- the length of x[n] is 1
- the length of h[n] is 1

(1) y[n] starts at = 0+(-3)=-3
(2) y[n] finishes at = 0+(-3)=-3
(3) the length of y[n] = 1+1-1=1

Example 5

Solution of first point (try to do at home as the previous one):

$$y[n] = 2h[n] \quad \text{or} \quad y[n] = -x[n+3]$$

$$y[-3] = -2, \qquad y[n] = 0 \quad \text{otherwise}$$

Example 5

Solution of second point:

$$y[n] = -x[n+3]$$

Example 6

Given:

$$x[0] = 2, \quad x[1] = 2, \qquad x[n] = 0 \quad \text{otherwise}$$

$$h[-3] = -1, \quad h[n] = 0 \quad \text{otherwise}$$

*** Obtain the output of the LTI system y[n], with impulse response h[n] Namely, y[n]=?**

*** Write the corresponding *linear difference equation* of this system (with constant coefficients and null initial conditions)**

Example 6

Then, in this example:

- start of x[n] at 0, start of h[n] at -3
- end of x[n] at 1, finish of h[n] at -3
- the length of x[n] is 2
- the length of h[n] is 1

(1) y[n] starts at = 0+(-3)=-3
(2) y[n] finishes at = 1+(-3)=-2
(3) the length of y[n] = 2+1-1=2

Example 6

Solution of first point (try to do at home as the previous one):

$$y[n] = 2h[n] + 2h[n-1] \quad \text{or} \quad y[n] = -x[n+3]$$

$$y[-3] = -2, \quad y[-2] = -2, \quad y[n] = 0 \quad \text{otherwise}$$

Example 6

Solution of second point: (again as Example 5)

$$y[n] = -x[n+3]$$

Example 7

Given:

$$x[0] = 2, \quad x[n] = 0 \quad \text{otherwise}$$
$$h[-3] = -1, \quad h[0] = 2, \quad h[n] = 0 \quad \text{otherwise}$$

*** Obtain the output of the LTI system y[n], with impulse response h[n]**
Namely, y[n]=?

*** Write the corresponding *linear difference equation* of this system (with constant coefficients and null initial conditions)**

Example 7

Then, in this example:

- start of x[n] at 0, start of h[n] at -3
- end of x[n] at 0, finish of h[n] at 0
- the length of x[n] is 1
- the length of h[n] is 4

(1) y[n] starts at = 0+(-3)=-3
(2) y[n] finishes at = 0+0=0
(3) the length of y[n] = 1+4-1=4

Example 7

Solution of first point (try to do at home as the previous one):

$$y[n] = 2h[n] \qquad \text{or} \qquad y[n] = -x[n+3] + 2x[n]$$

$$y[-3] = -2, \quad y[-2] = 0, \quad y[-1] = 0,$$
$$y[0] = 4, \quad y[n] = 0 \quad \text{otherwise}$$

Example 7

Solution of second point:

$$y[n] = -x[n+3] + 2x[n]$$

Example 8

NOW WE CONSIDER SIGNALS with INFINITE LENGTHS

Given:

$$x[n] = a^n u[n]$$

$$h[n] = b^n u[n] \qquad \text{with } a \neq b$$

Obtain the output of the LTI system y[n], with impulse response h[n]

Namely, y[n]=?

Example 8

We know that:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

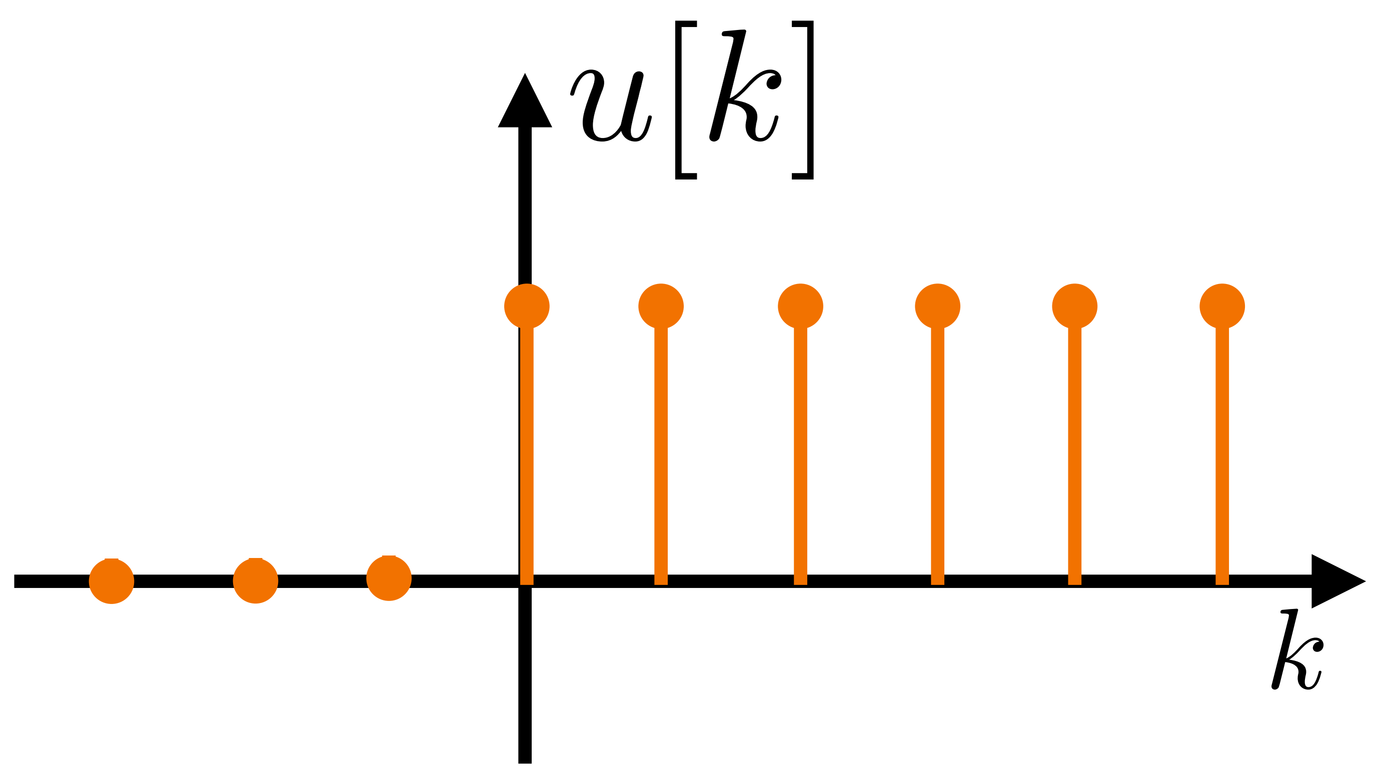
$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{+\infty} h[k]x[n-k]$$

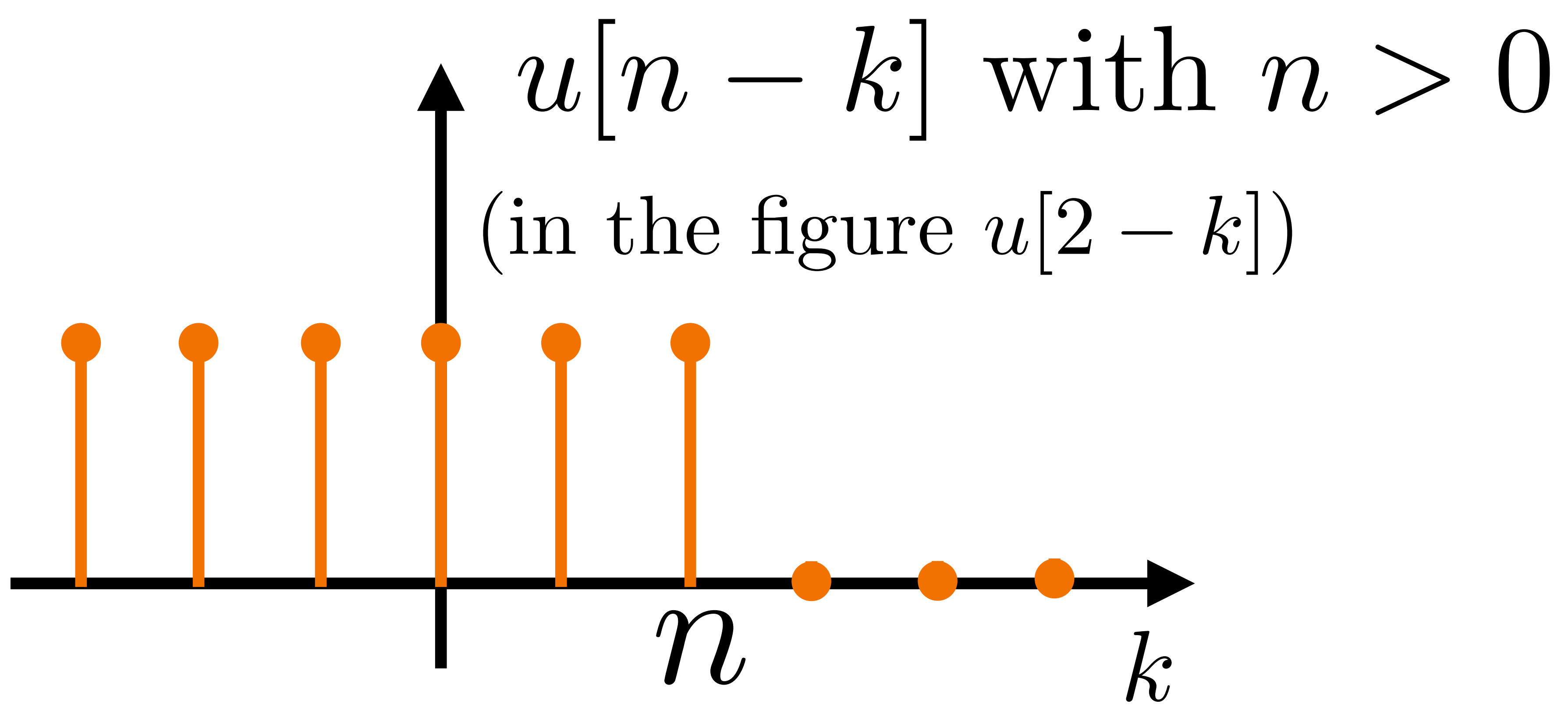
Example 8

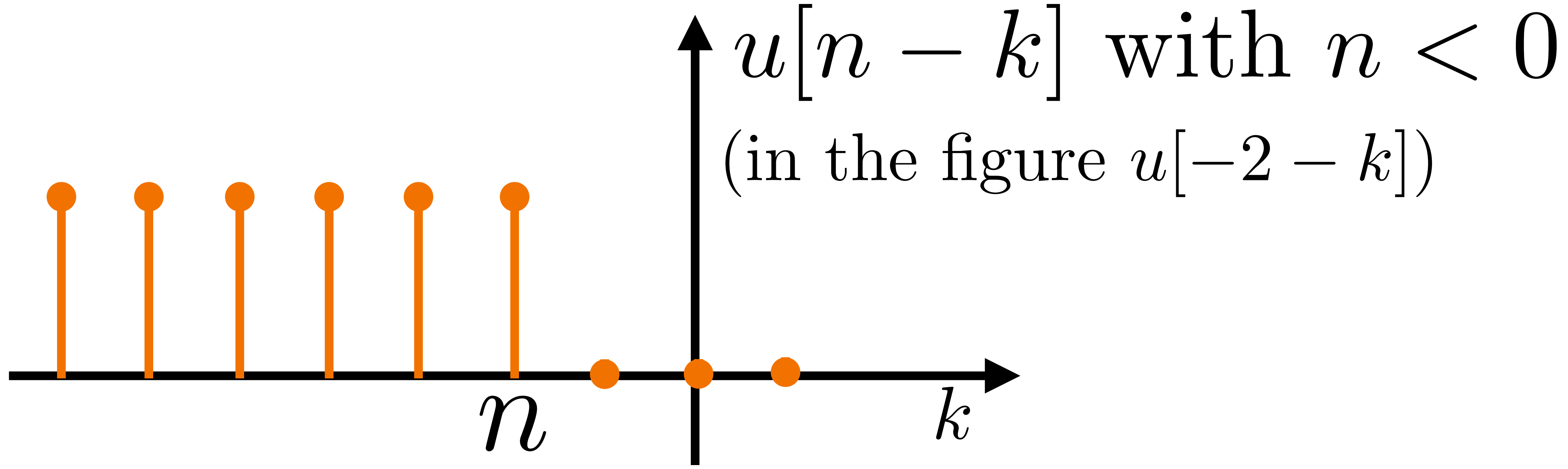
Then we can write :

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

$$y[n] = \sum_{k=-\infty}^{+\infty} a^k u[k] b^{n-k} u[n-k]$$

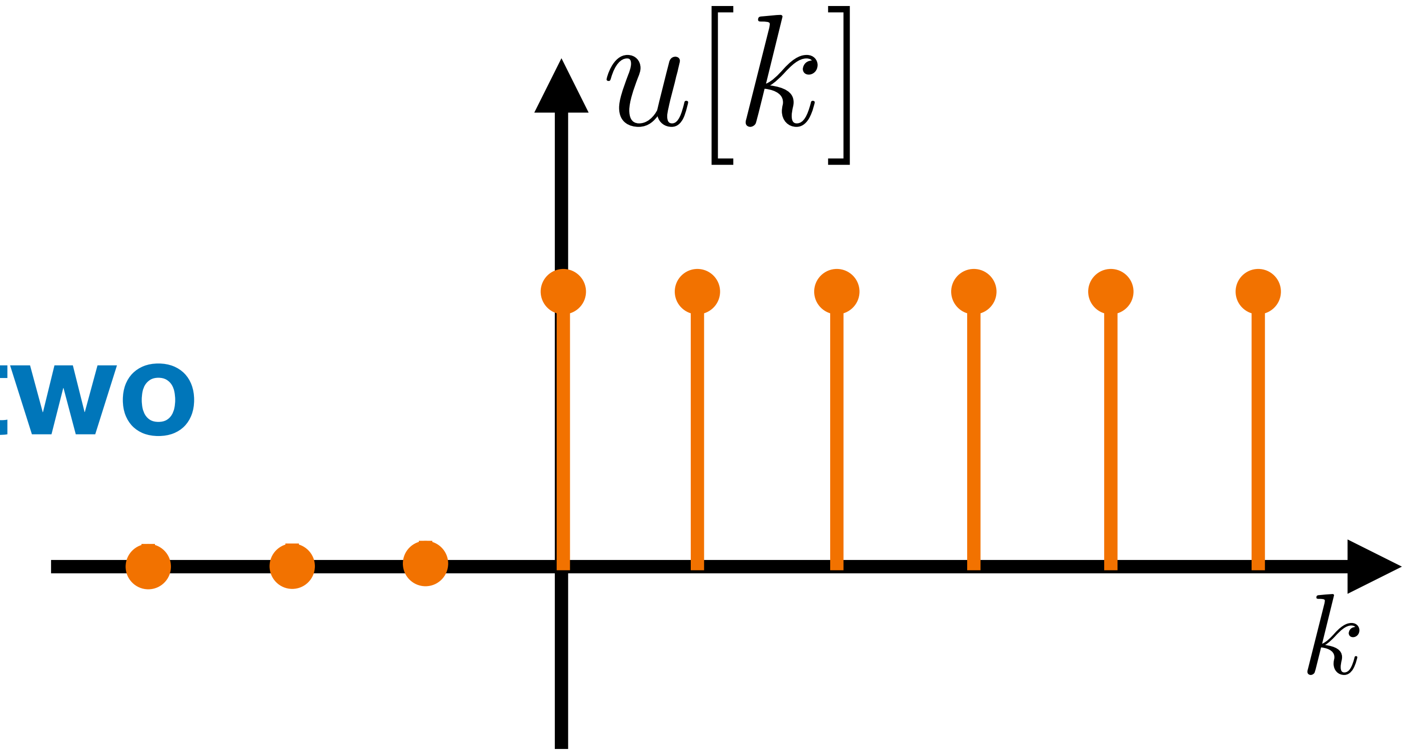


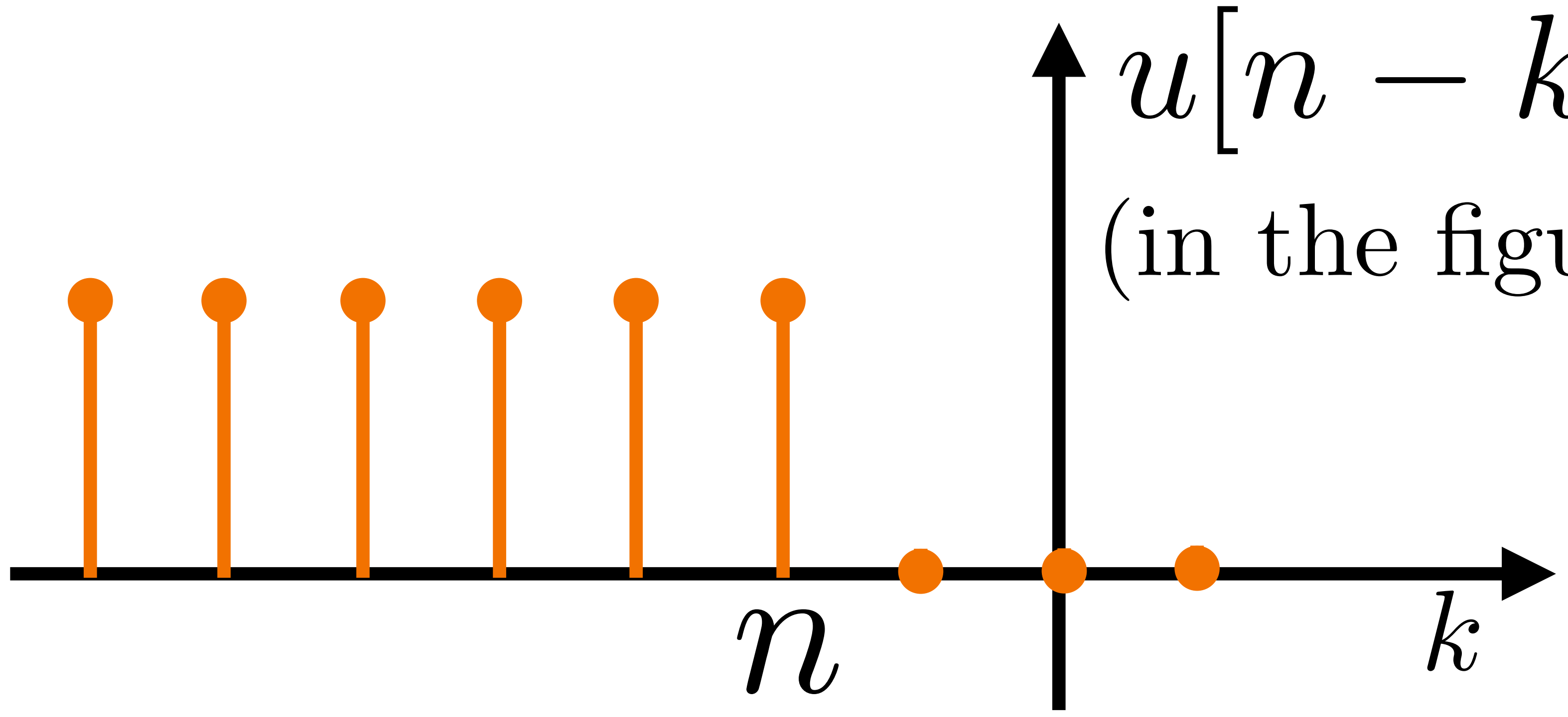




Example 8

Looking this two figures:





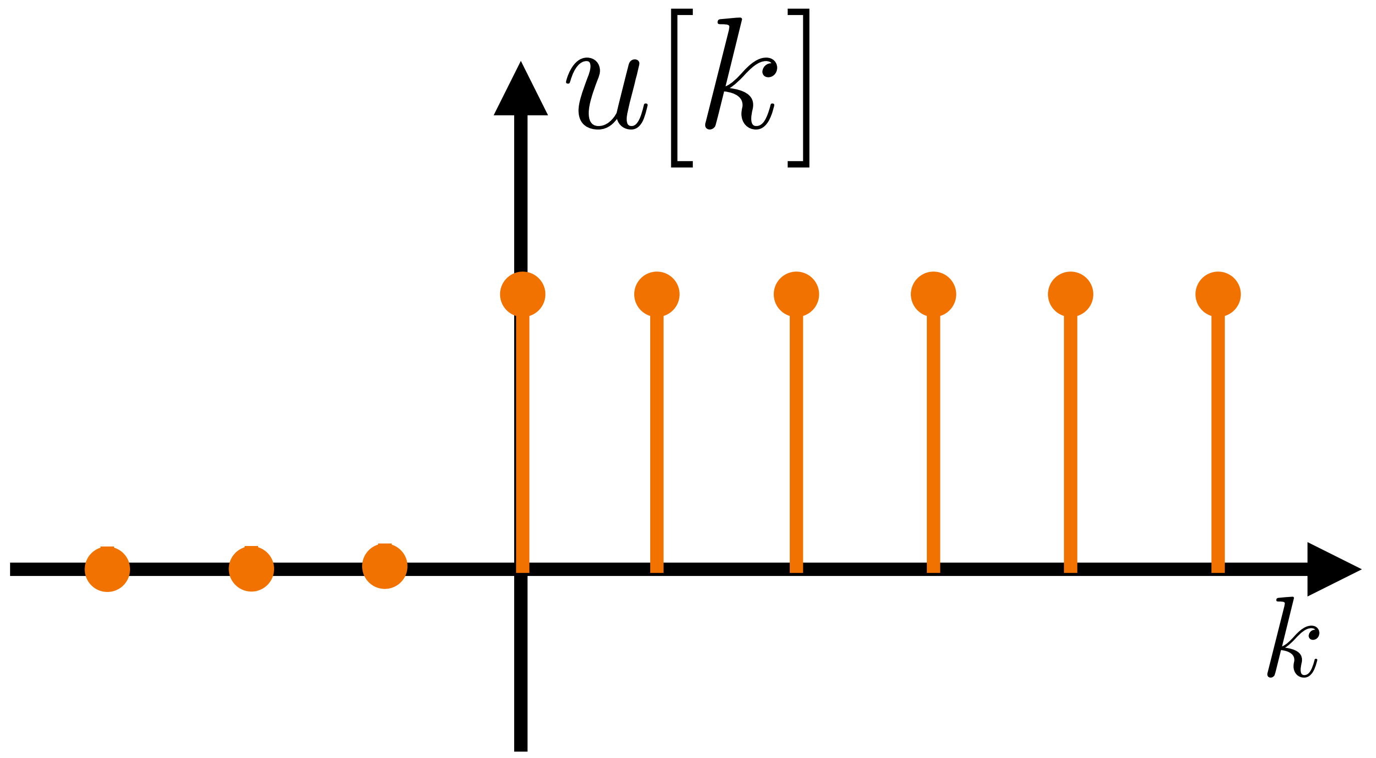
We have:

$$y[n] = \sum_{k=-\infty}^{+\infty} a^k b^{n-k} u[k]u[n-k]$$

$$y[n] = 0 \text{ for all } n < 0$$

Example 8

Looking this :



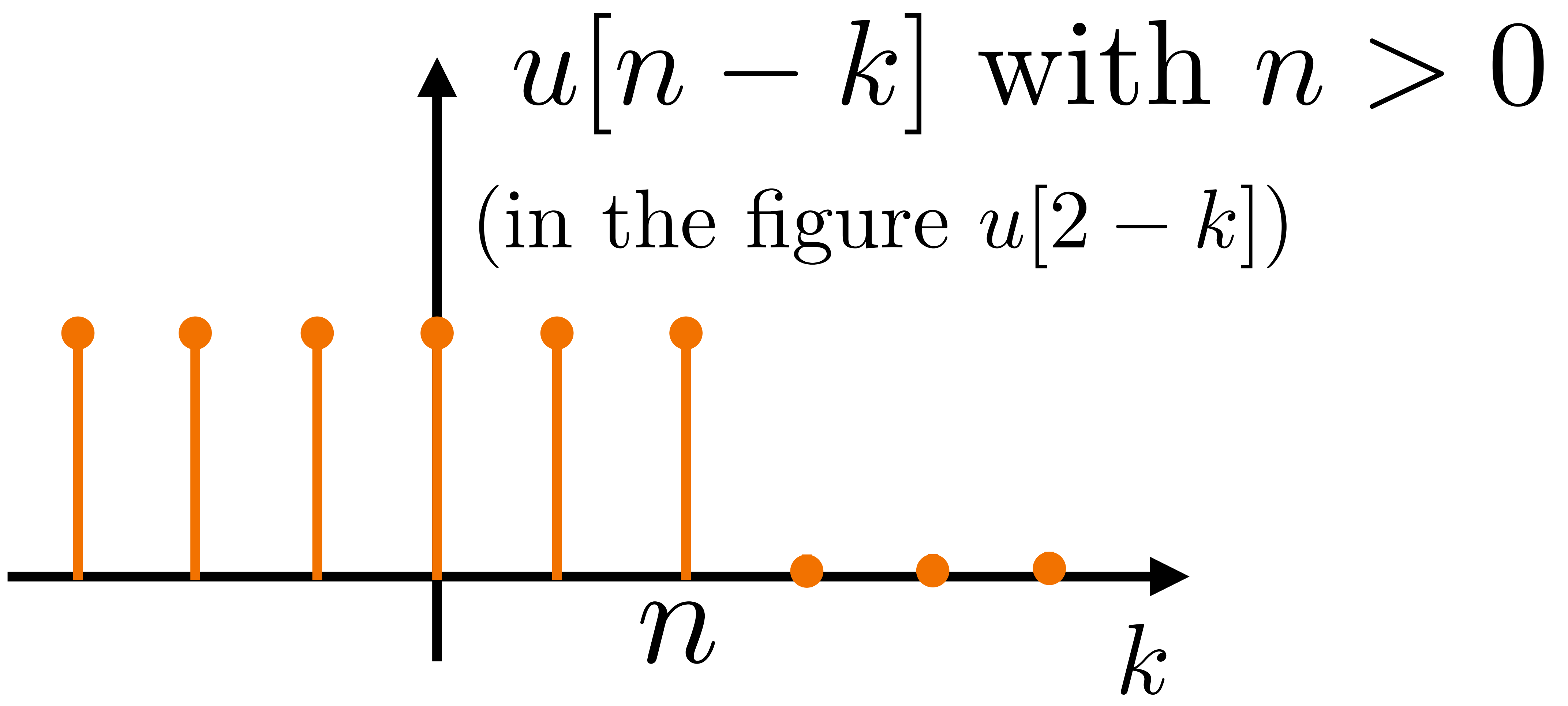
We have:

$$y[n] = \sum_{k=-\infty}^{+\infty} a^k b^{n-k} u[k] u[n-k]$$

$$y[n] = \sum_{k=0}^{+\infty} a^k b^{n-k} u[n-k]$$

Example 8

Looking this :



We have:

$$y[n] = \sum_{k=0}^{+\infty} a^k b^{n-k} u[n-k]$$

$$y[n] = \sum_{k=0}^{n} a^k b^{n-k}$$

Example 8

With more rearrangements:

$$y[n] = \sum_{k=0}^{n} a^k b^{n-k}$$

$$y[n] = \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k b^n$$

$$y[n] = b^n \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k$$

Example 8

Recalling: $$\sum_{k=0}^{n} r^k = \frac{1-r^{n+1}}{1-r}$$

with: $$r = \frac{a}{b}$$

We finally obtain: $$y[n] = b^n \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k = b^n \frac{1-\left(\frac{a}{b}\right)^{n+1}}{1-\left(\frac{a}{b}\right)}$$

Example 8

This is ALMOST the solution: $$y[n] = b^n \frac{1-\left(\frac{a}{b}\right)^{n+1}}{1-\left(\frac{a}{b}\right)}$$

with some rearrangements:
(just for make it more "pretty")

$$y[n] = \frac{b^n - a^{n+1}\frac{1}{b}}{\frac{b-a}{b}}$$

$$y[n] = \frac{b^{n+1} - a^{n+1}}{b-a}$$

Example 8

The complete solution is:

$$\begin{cases} y[n] = \dfrac{b^{n+1} - a^{n+1}}{b - a} & n \geq 0 \\ y[n] = 0 & n < 0 \end{cases}$$

That can be summarized as:

$$y[n] = \frac{b^{n+1} - a^{n+1}}{b - a} u[n]$$

Note that with a=b… we have a indeterminate form….

Example 9

NOW WE CONSIDER SIGNALS with INFINITE LENGTHS

Given:

$$x[n] = a^n u[n]$$

$$h[n] = a^n u[n]$$

Obtain the output of the LTI system y[n], with impulse response h[n]

Namely, y[n]=?

Example 9

Following Example 8, we can easily arrive to:

$$y[n] = 0 \text{ for all } n < 0$$

and:

$$y[n] = \sum_{k=0}^{n} a^k a^{n-k}$$

$$y[n] = a^n \sum_{k=0}^{n} 1$$

Example 9

Then, thinking to sum (n+1)-times "1": $$y[n] = a^n \sum_{k=0}^{n} 1$$

$$y[n] = a^n \underbrace{(1 + 1 + 1 ... + 1)}_{(n+1)-times}$$

$$y[n] = a^n (n + 1)$$

Example 9

The complete solution is:

$$\begin{cases} y[n] = a^n(n+1) & n \geq 0 \\ y[n] = 0 & n < 0 \end{cases}$$

That can be summarized as:

$$y[n] = a^n(n+1)u[n]$$

Questions?