STUDY of a filter: IIR - AR(1)

Discrete Time Systems

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AR(1)

$$y[n] = by[n-1] + x[n] \qquad n \geq 0 \qquad b \in \mathbb{R}$$

Frequency Analysis (Transformed domain analysis) - and Stability

$$Y(z) - bz^{-1}Y(z) = X(z)$$

$$Y(z)(1 - bz^{-1}) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - bz^{-1}} = \frac{z}{z - b}$$

$$y[n] = by[n-1] + x[n] \qquad n \geq 0 \qquad b \in \mathbb{R}$$

AR(1)

$$H(z) = \frac{z}{z-b}$$

Zeros: z=0

Poles: z=b

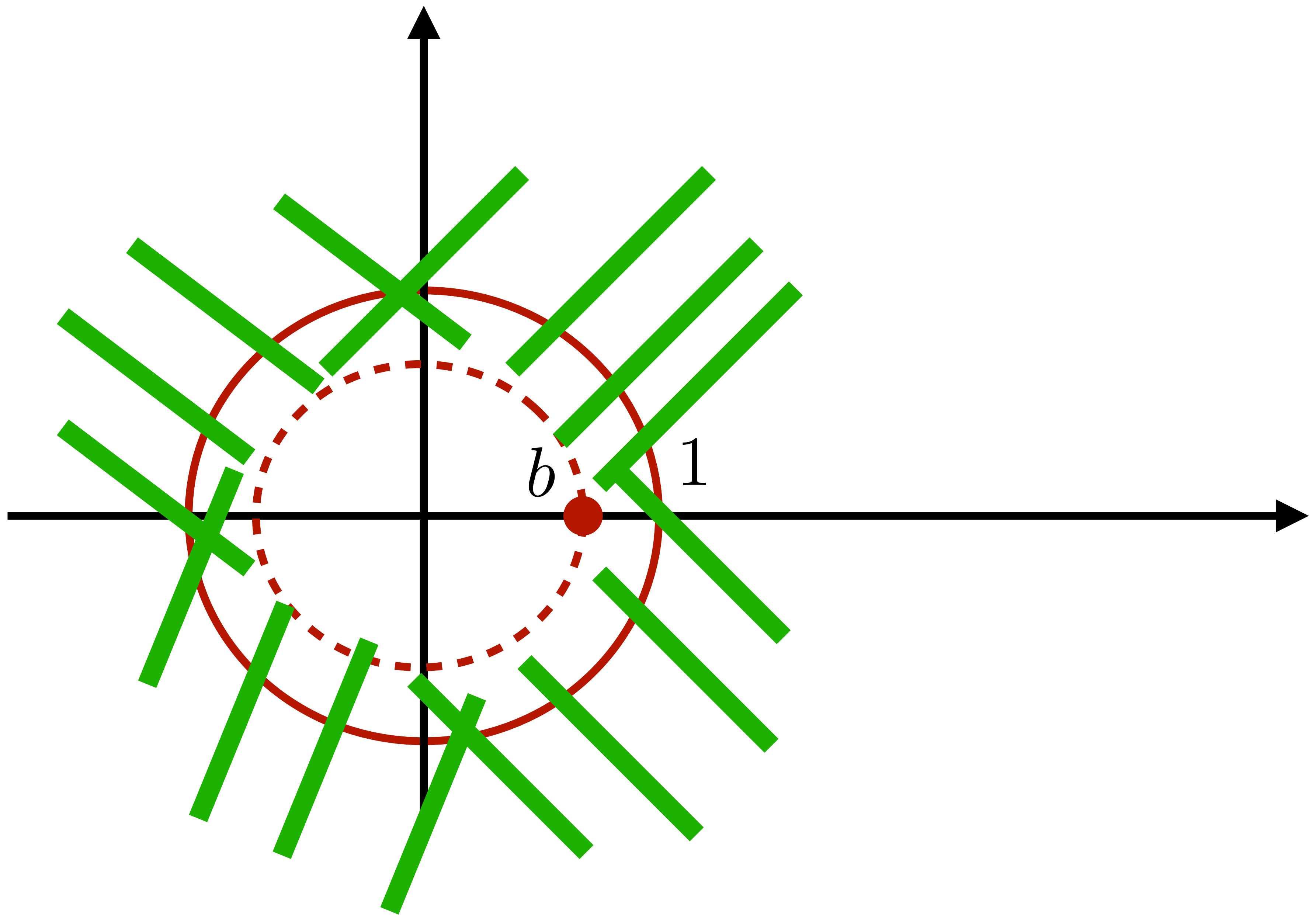
HOWEVER, since the system is causal, this is equivalent to say that h[n] must be a right-sided sequence

$$\text{ROC-2} ==> |z| > |b|$$

$$h_2[n] = b^n u[n]$$

right-sided

(for having the Standard Fourier Transform we need |b|<1)



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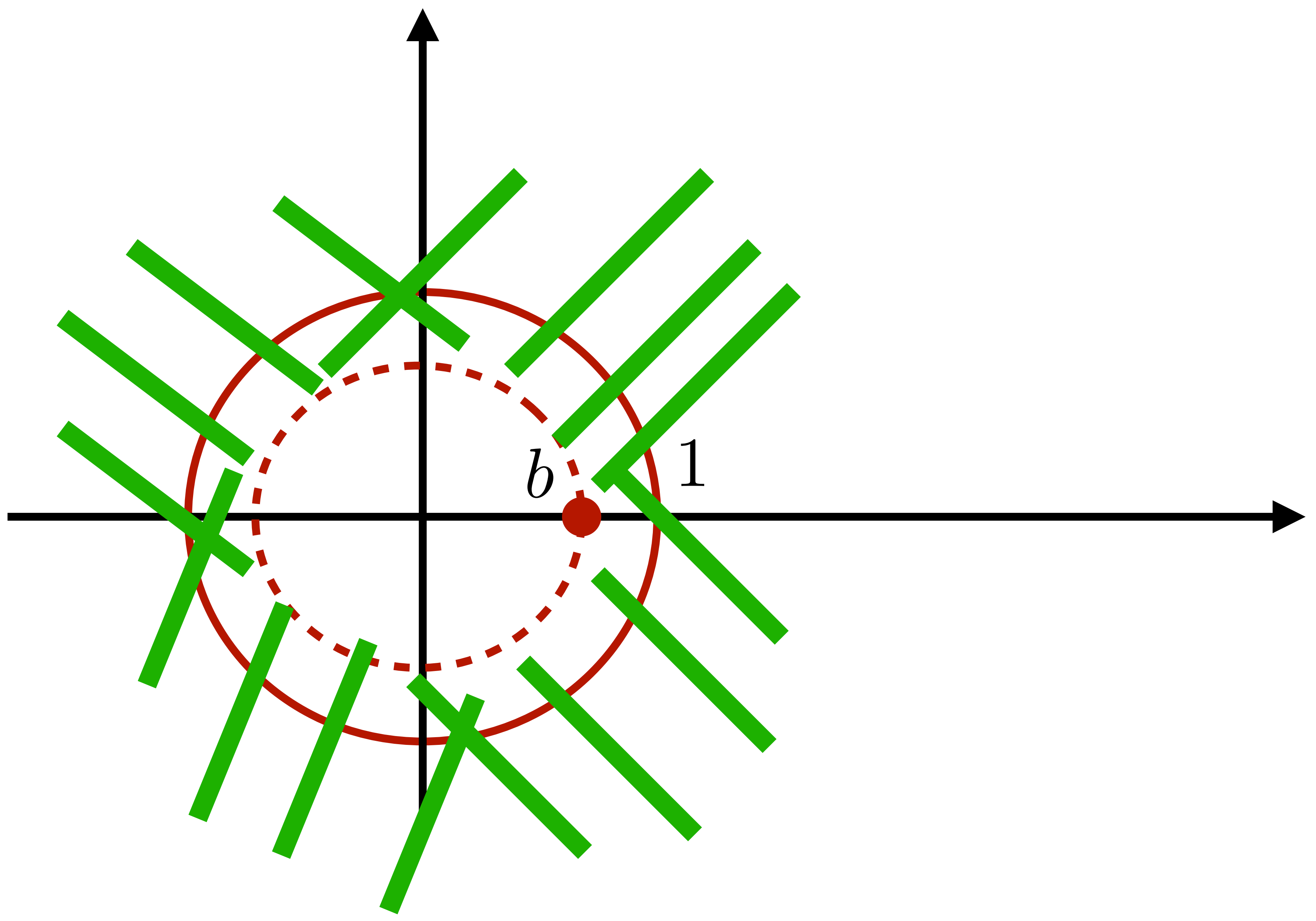
Since the system is **causal:**



-Standard Fourier Transform: **if |b|<1**

-Stability: **if |b|<1, poles inside circle of radius 1.**

- Generally, we have stability if h[n] has Standard Fourier Transform

$$y[n] = by[n-1] + x[n] \qquad n \geq 0 \qquad b \in \mathbb{R}$$

We see by recursion if what we write is true…

to find some values of h[n] recall that x[n]=delta[n]:

$$y[0] = by[-1] + x[0] = 0 + 1 = 1,$$

null initial conditions ==> for LTI systems !

When $x[n] = \delta[n]$ then $y[n] = h[n]$.

$$y[1] = by[0] + x[1] = b + 0 = b,$$

$$y[2] = by[1] + x[2] = b^2 + 0 = b^2,$$

$$y[3] = by[2] + x[3] = b^3 + 0 = b^3,$$

$$\ldots$$

$$y[k] = by[k-1] + x[k] = b^k + 0 = b^k,$$

$$\ldots$$

$$y[n] = by[n-1] + x[n] = b^n + 0 = b^n$$

$$h[n] = b^n u[n]$$

$$\rightarrow y[n] = b^n u[n]$$

as expected!!

$$y[n] = by[n-1] + x[n] \qquad n \geq 0 \qquad b \in \mathbb{R}$$

(Forced) solution of the difference equation

$$y[n] = b^n u[n] \qquad h[n] = b^n u[n]$$

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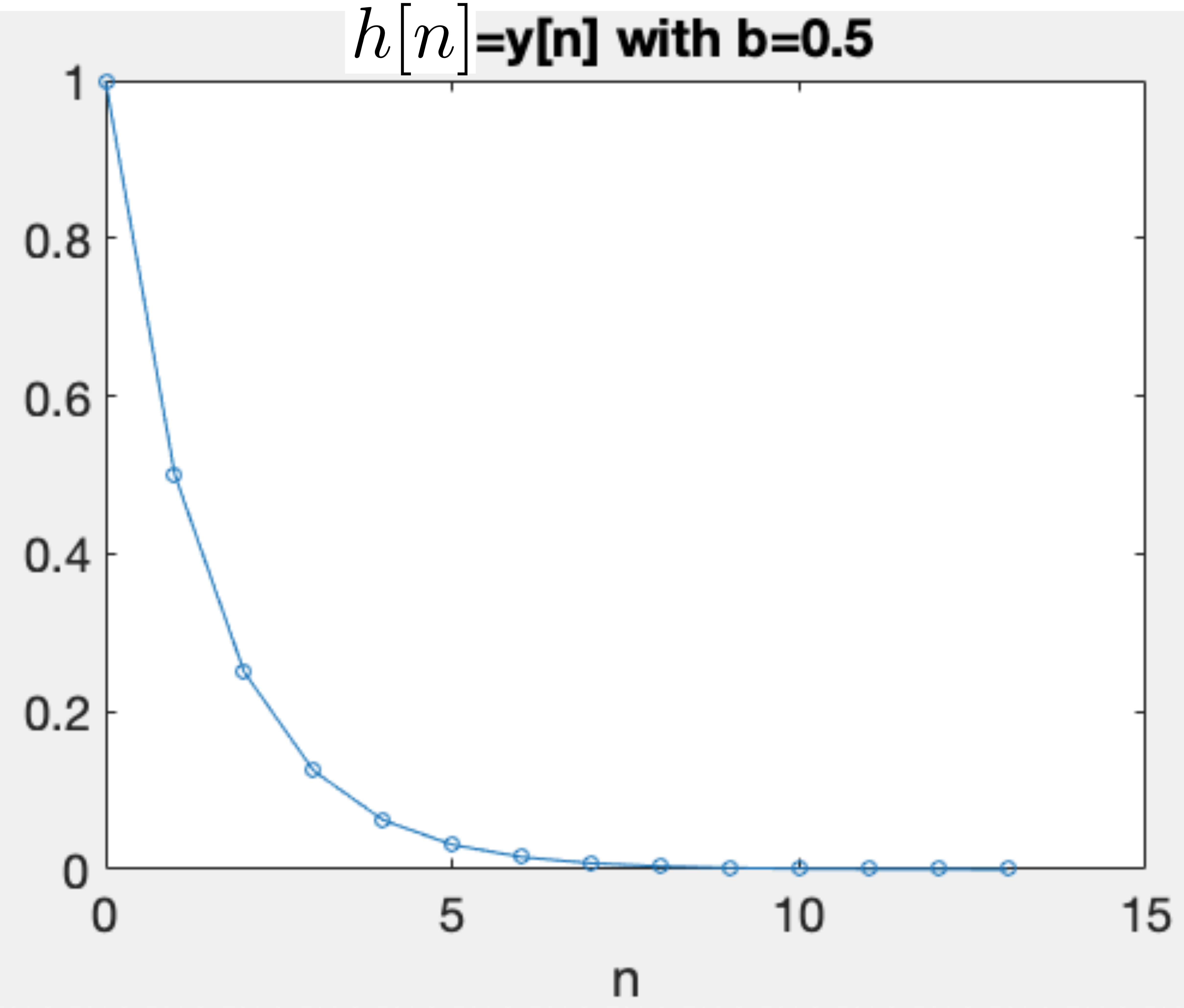

h[n] with Standard FT System ==> STABLE

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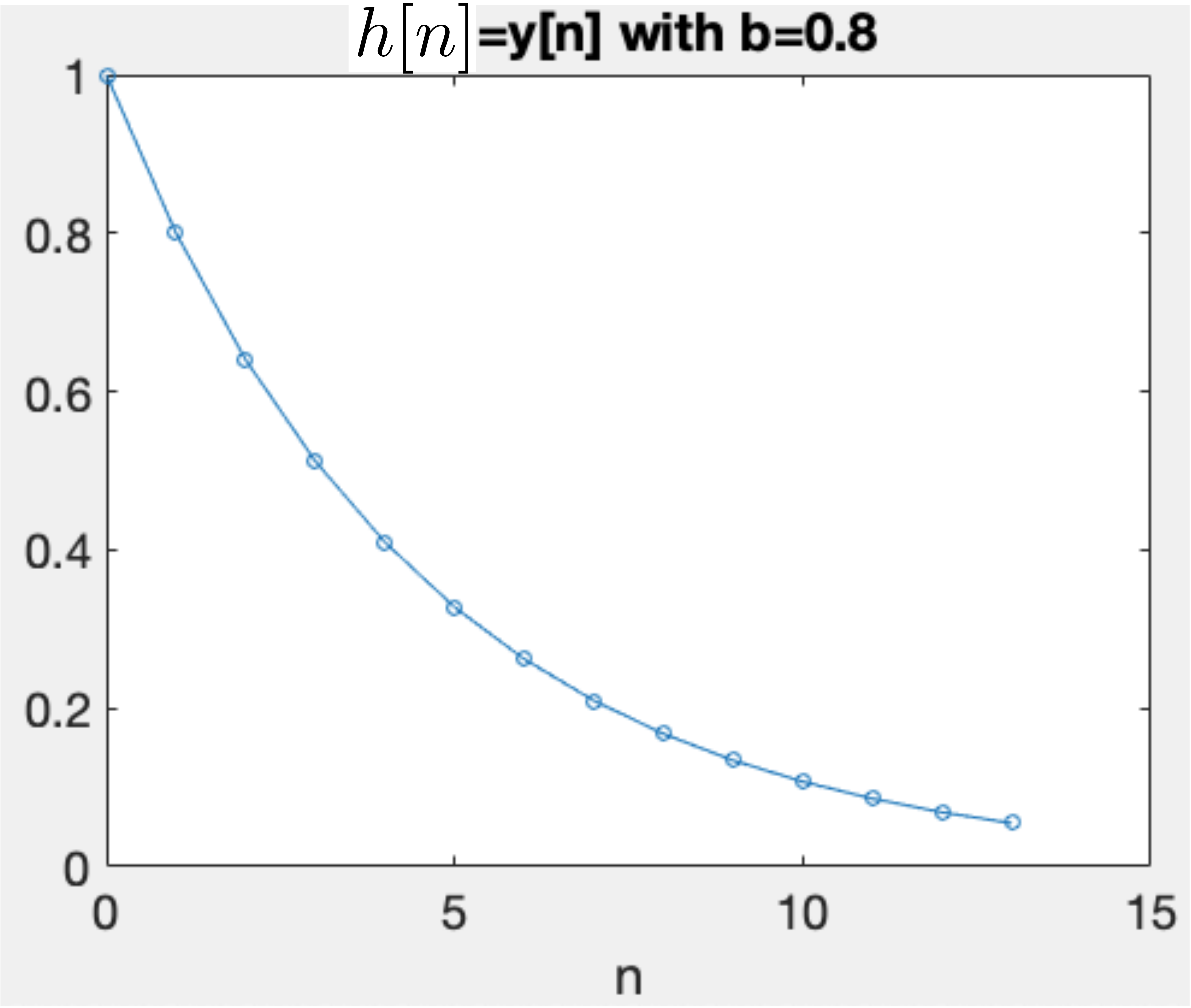

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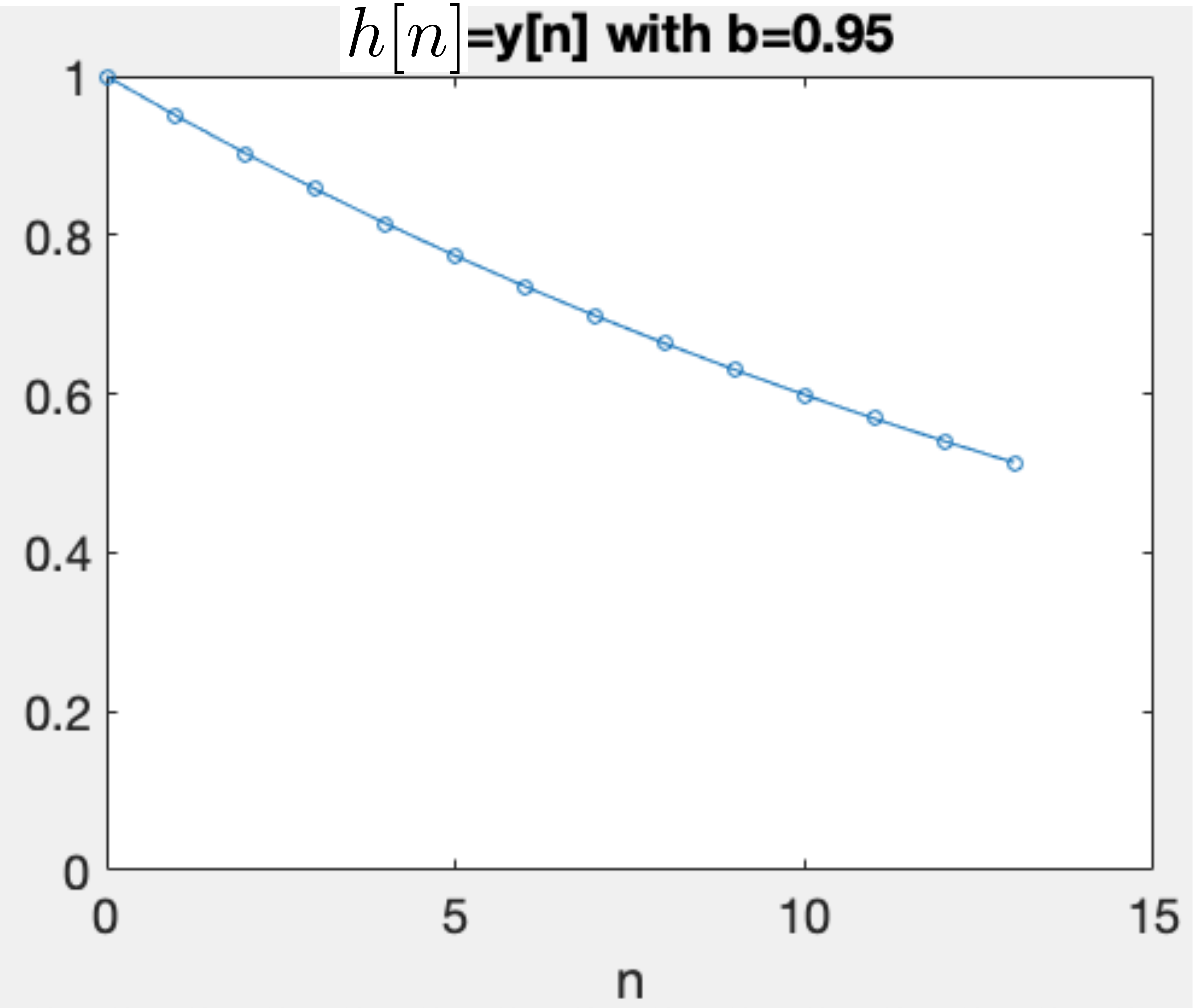

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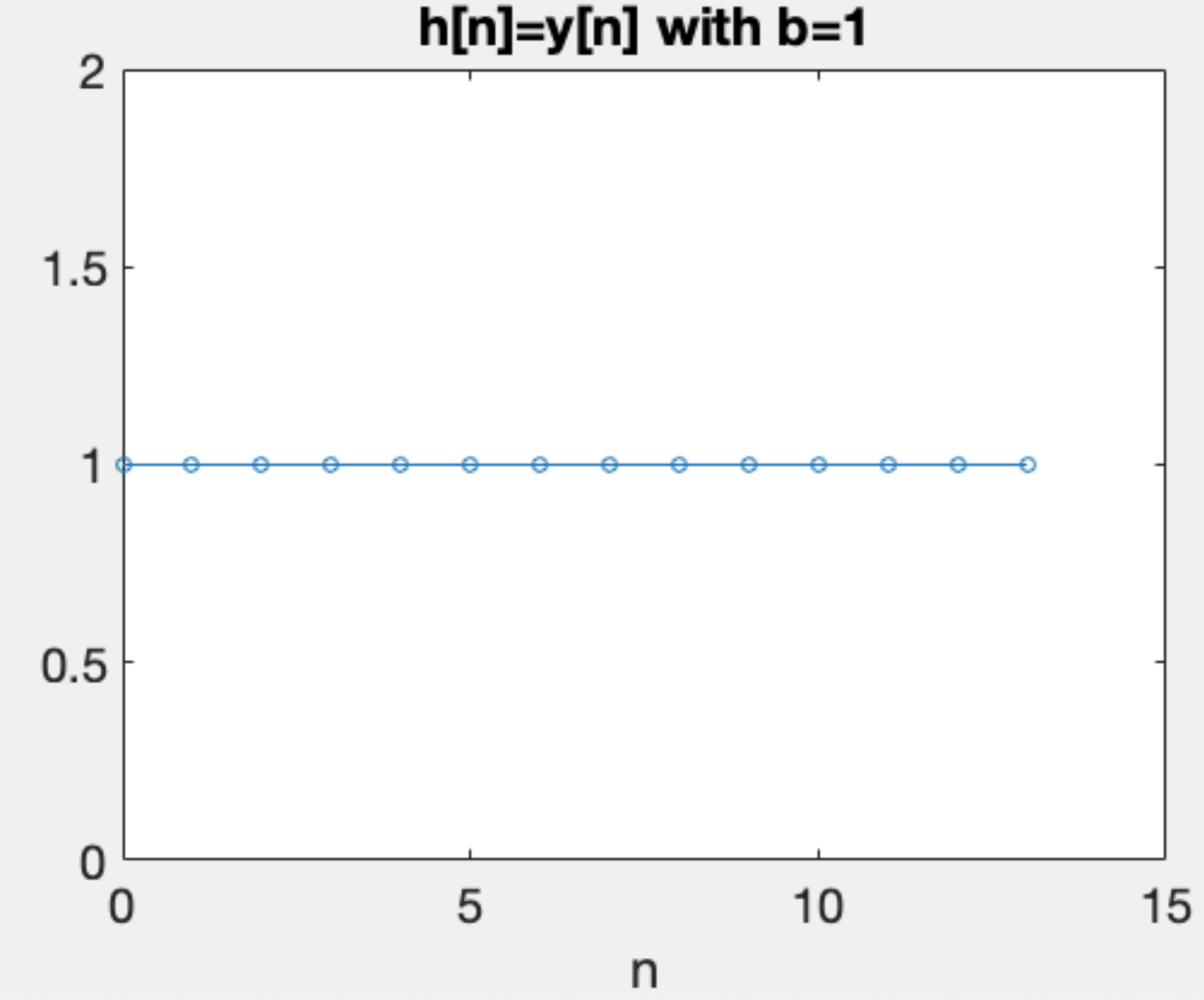
h[n] HAS NOT a Standard FT System ==> UNSTABLE (limit of stability, pole at b=1)



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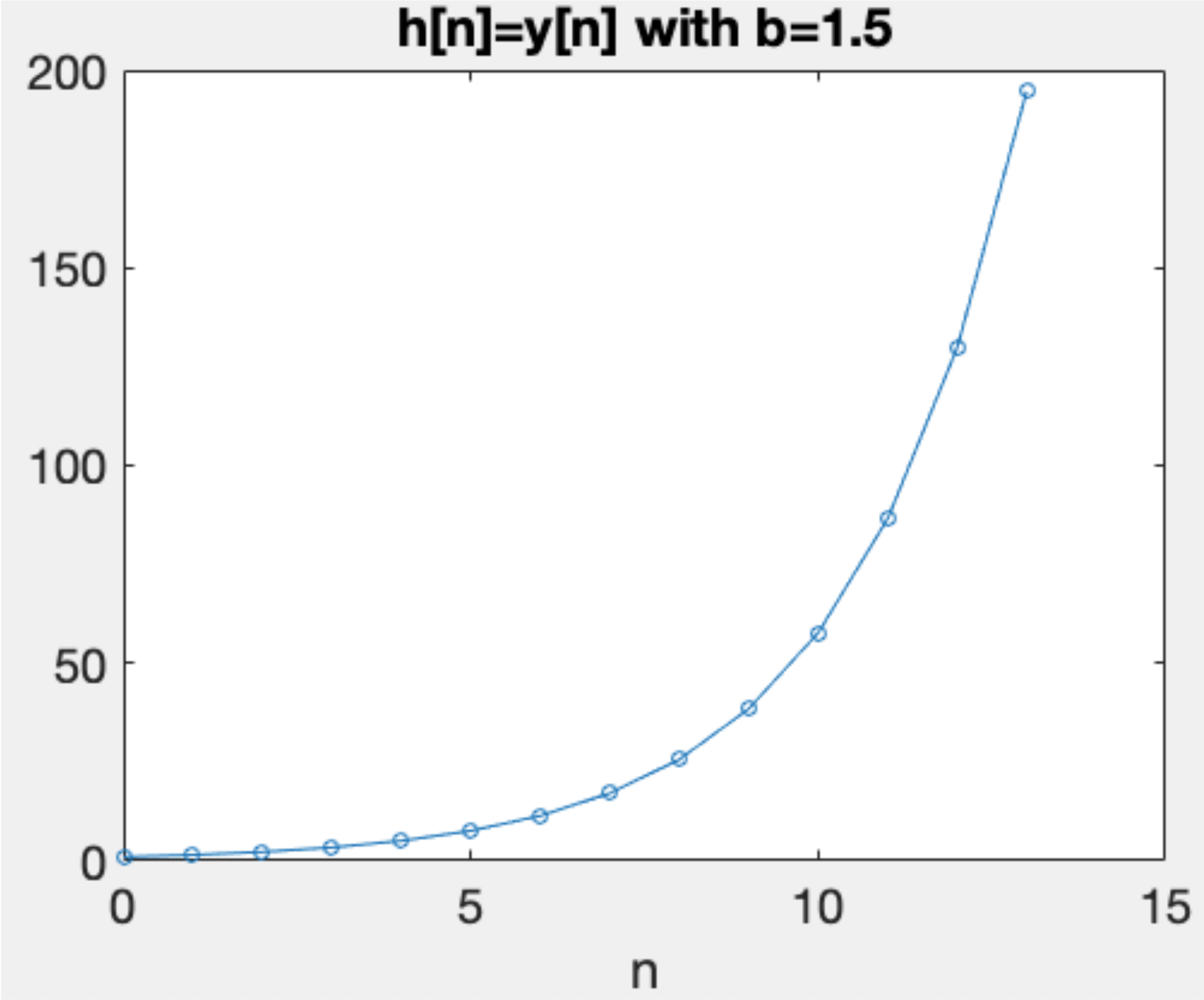

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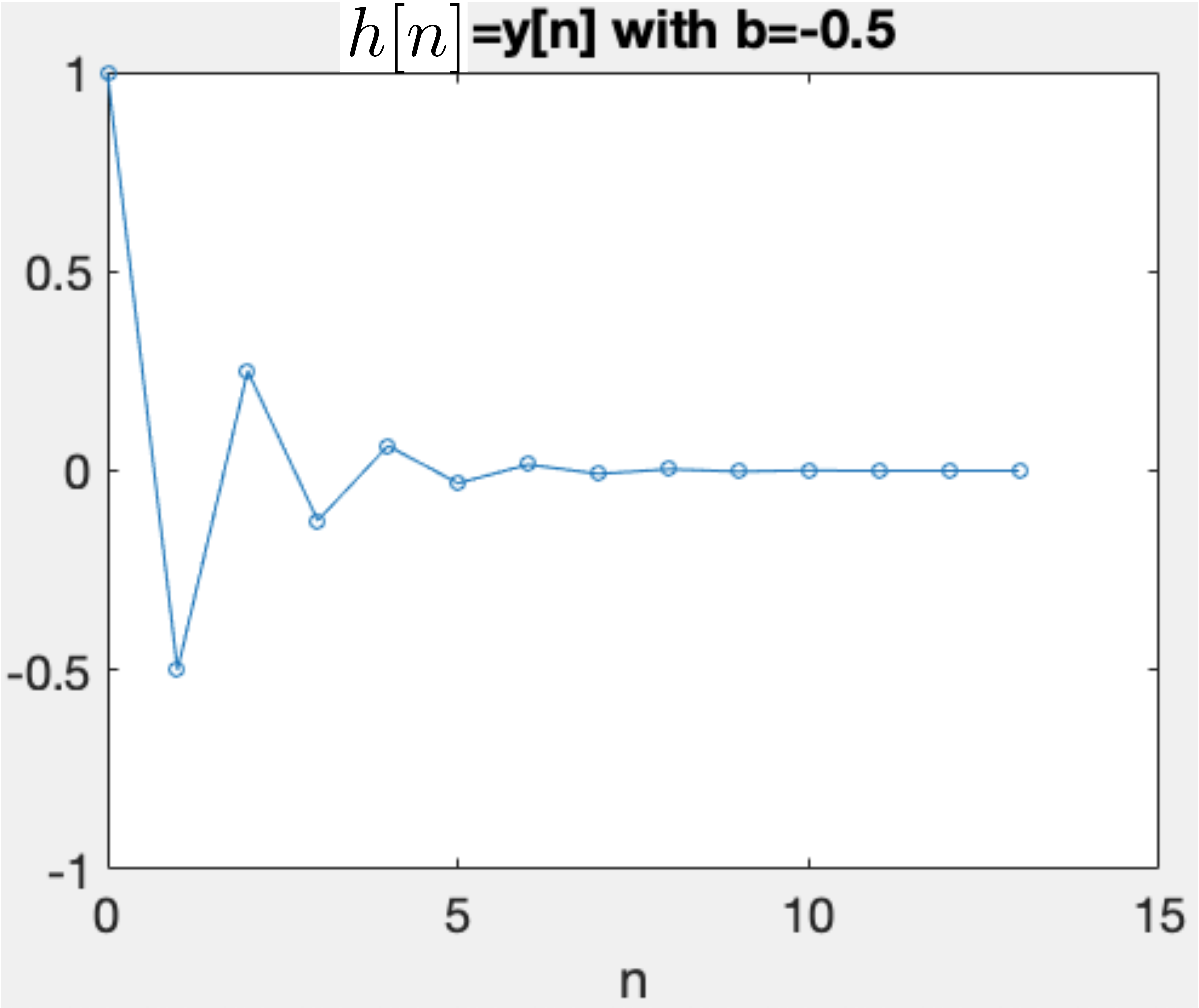

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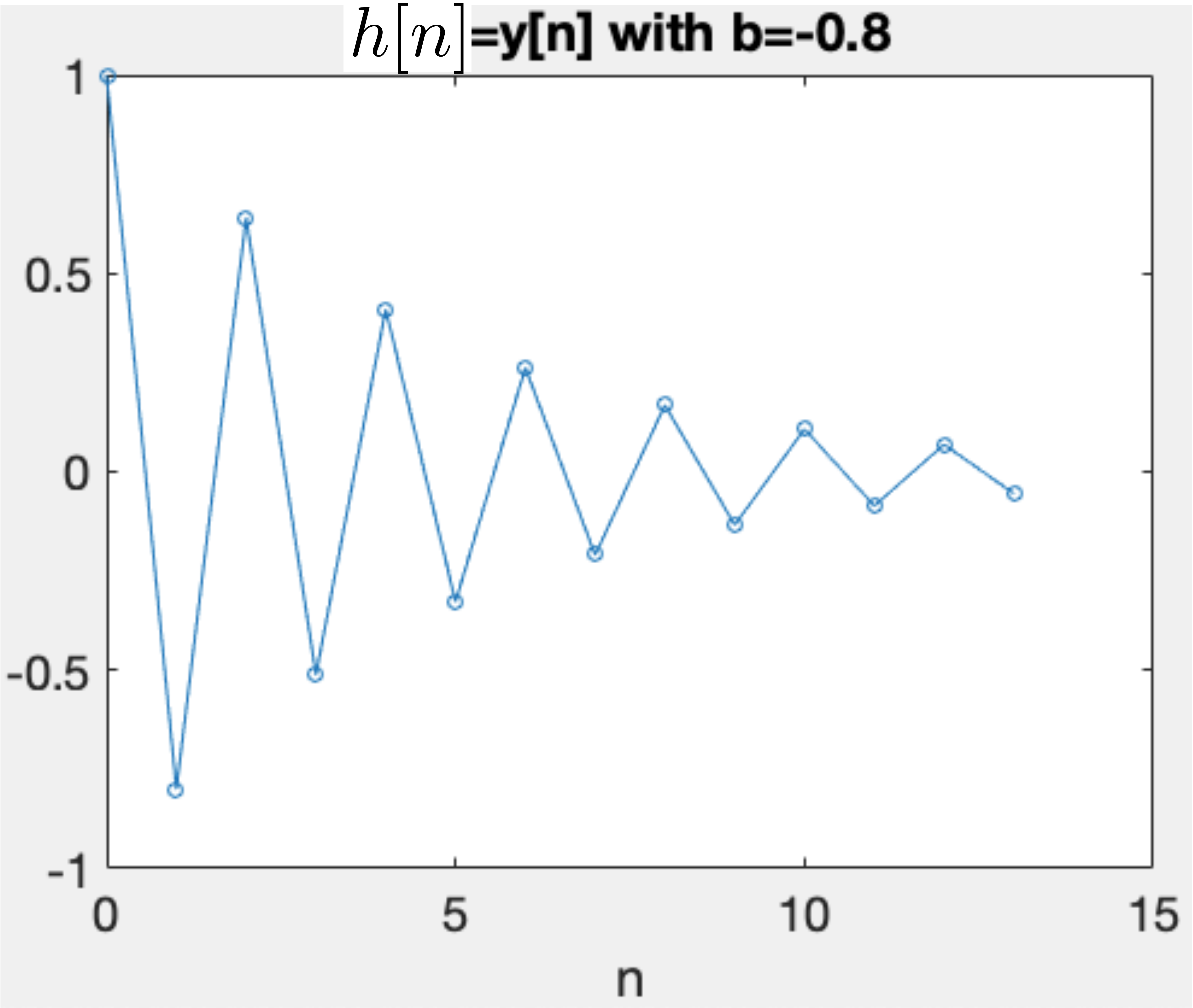

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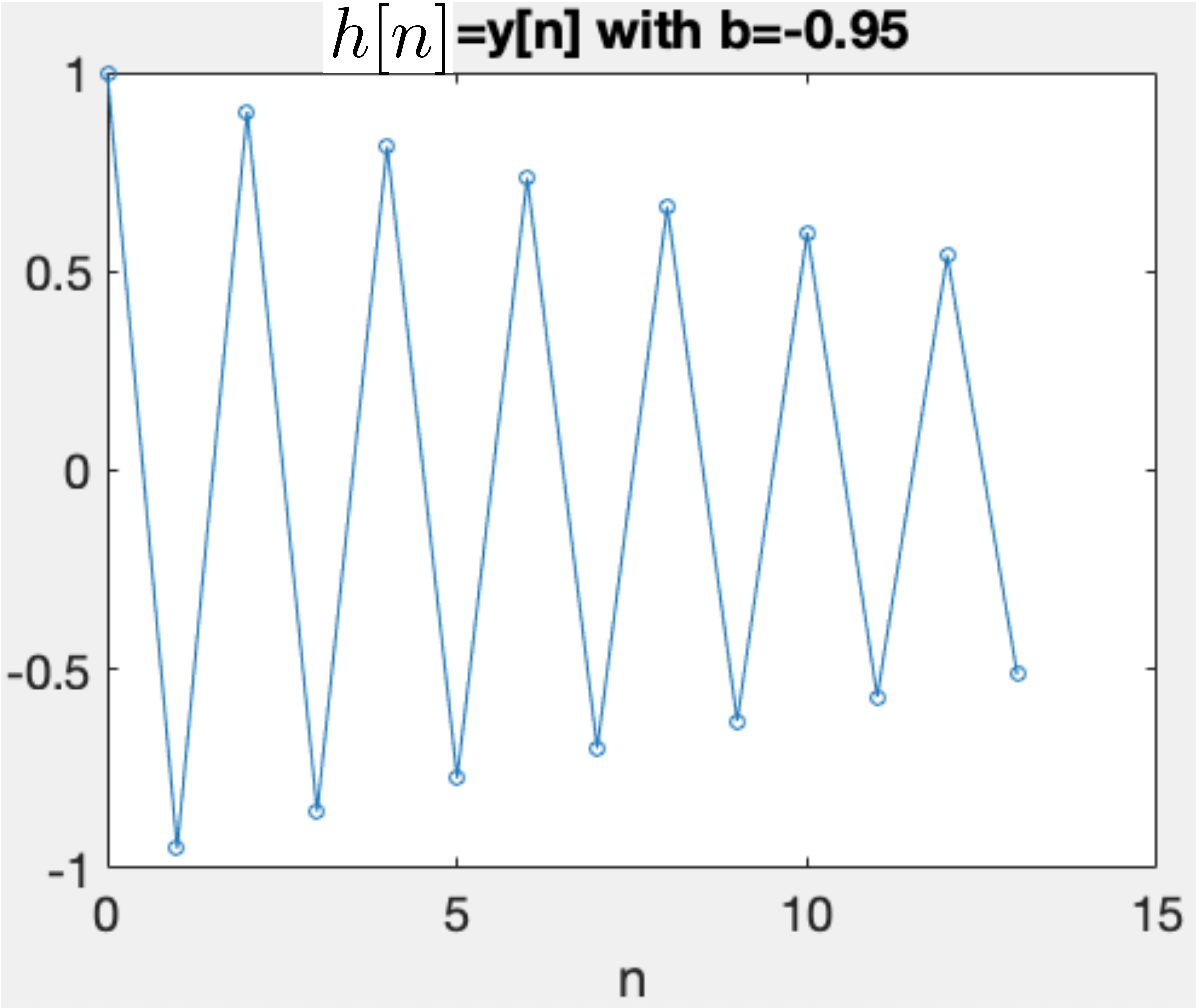

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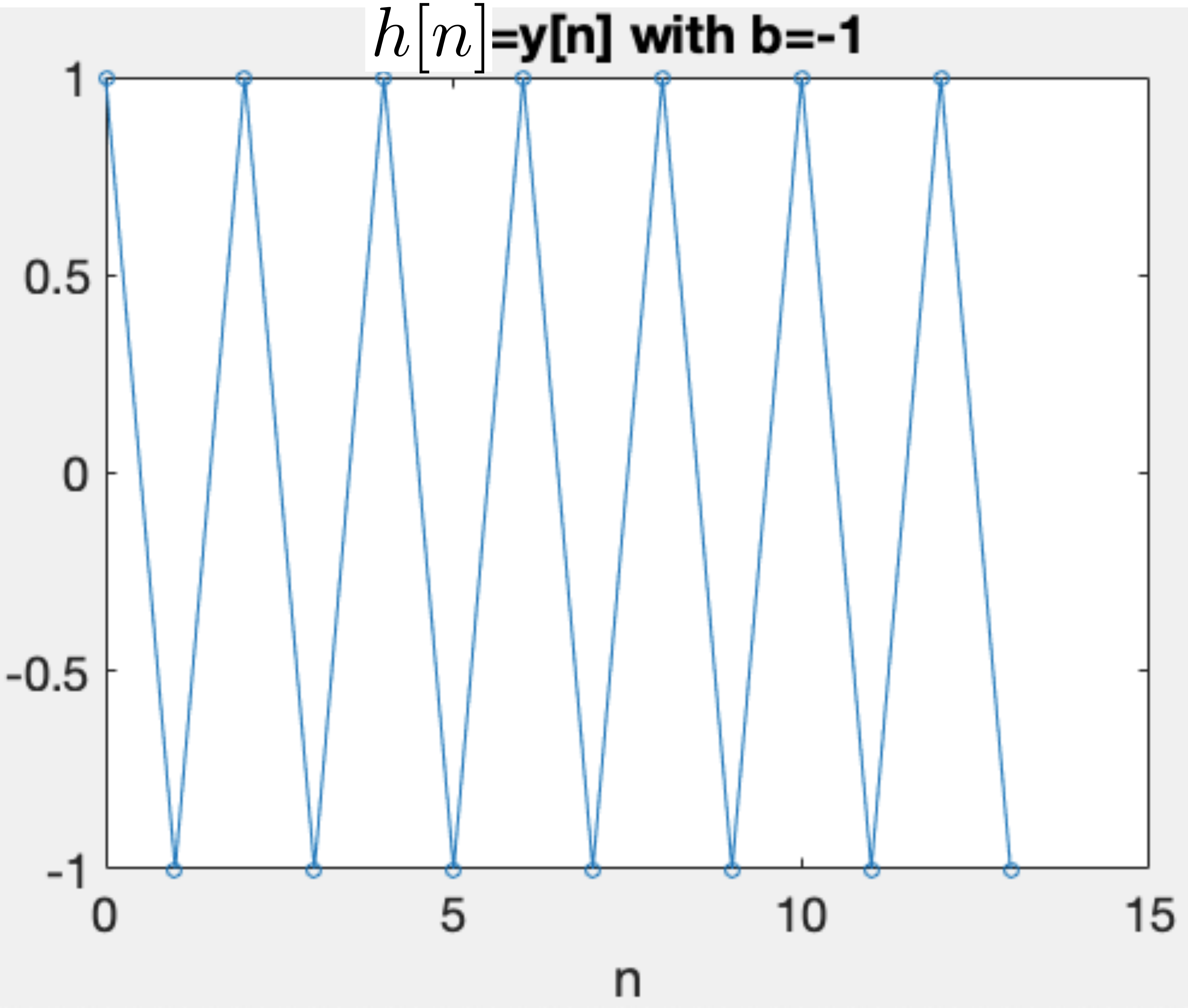

h[n] HAS NOT a Standard FT System ==> UNSTABLE (limit of stability, pole at b=-1)

$$y[n] = by[n-1] + x[n] \quad n \geq 0 \quad b \in \mathbb{R}$$

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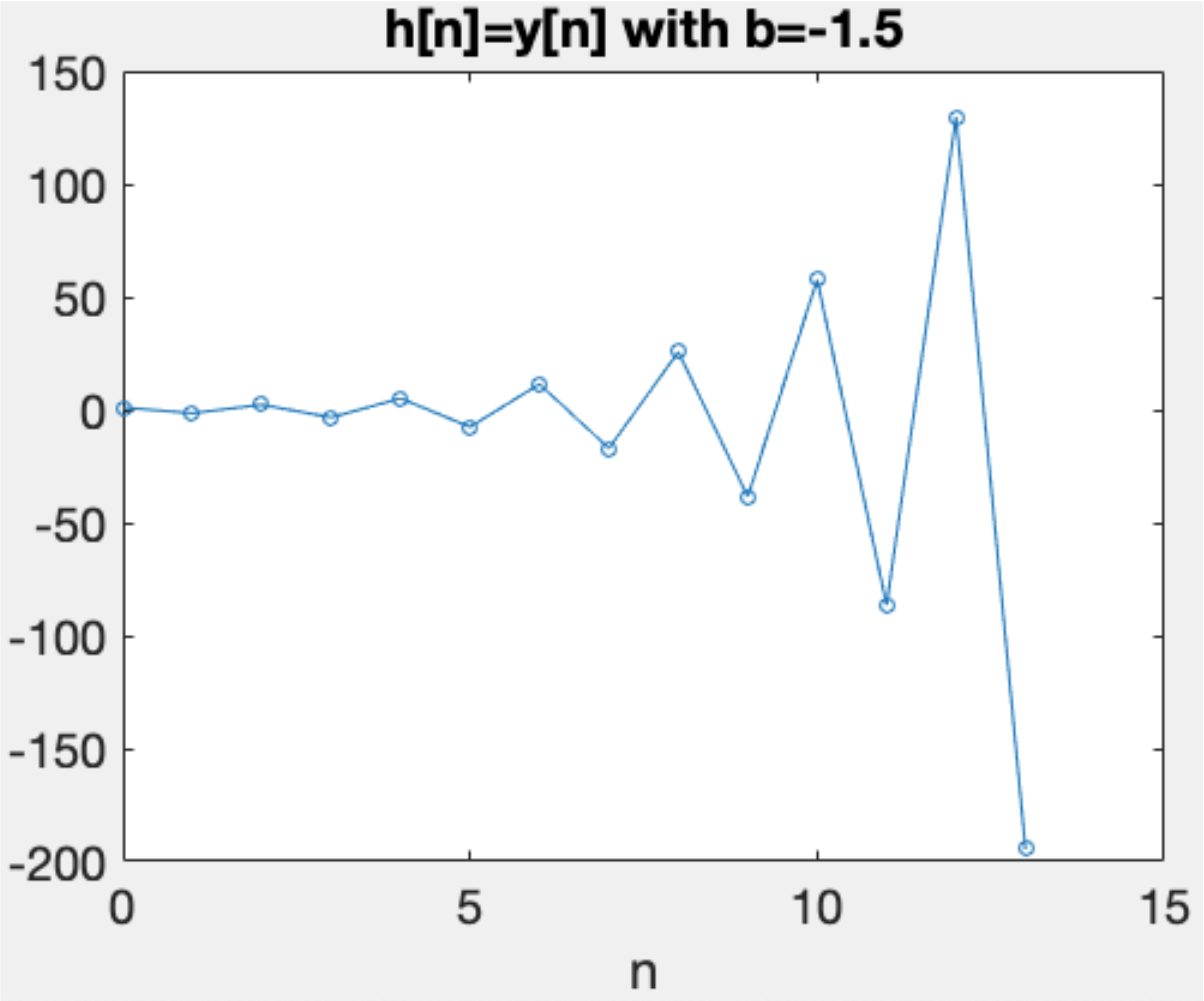

AR(1)

$$y[n] = by[n-1] + x[n] \qquad n \geq 0 \qquad b \in \mathbb{R}$$

$$H(z) = \frac{z}{z-b} \longrightarrow H(\Omega) = \frac{e^{j\Omega}}{e^{j\Omega} - b}$$

Standard Fourier Transform
of h[n]

When |b|<1 we h[n] has a
Standard FT, i.e., H(Omega)

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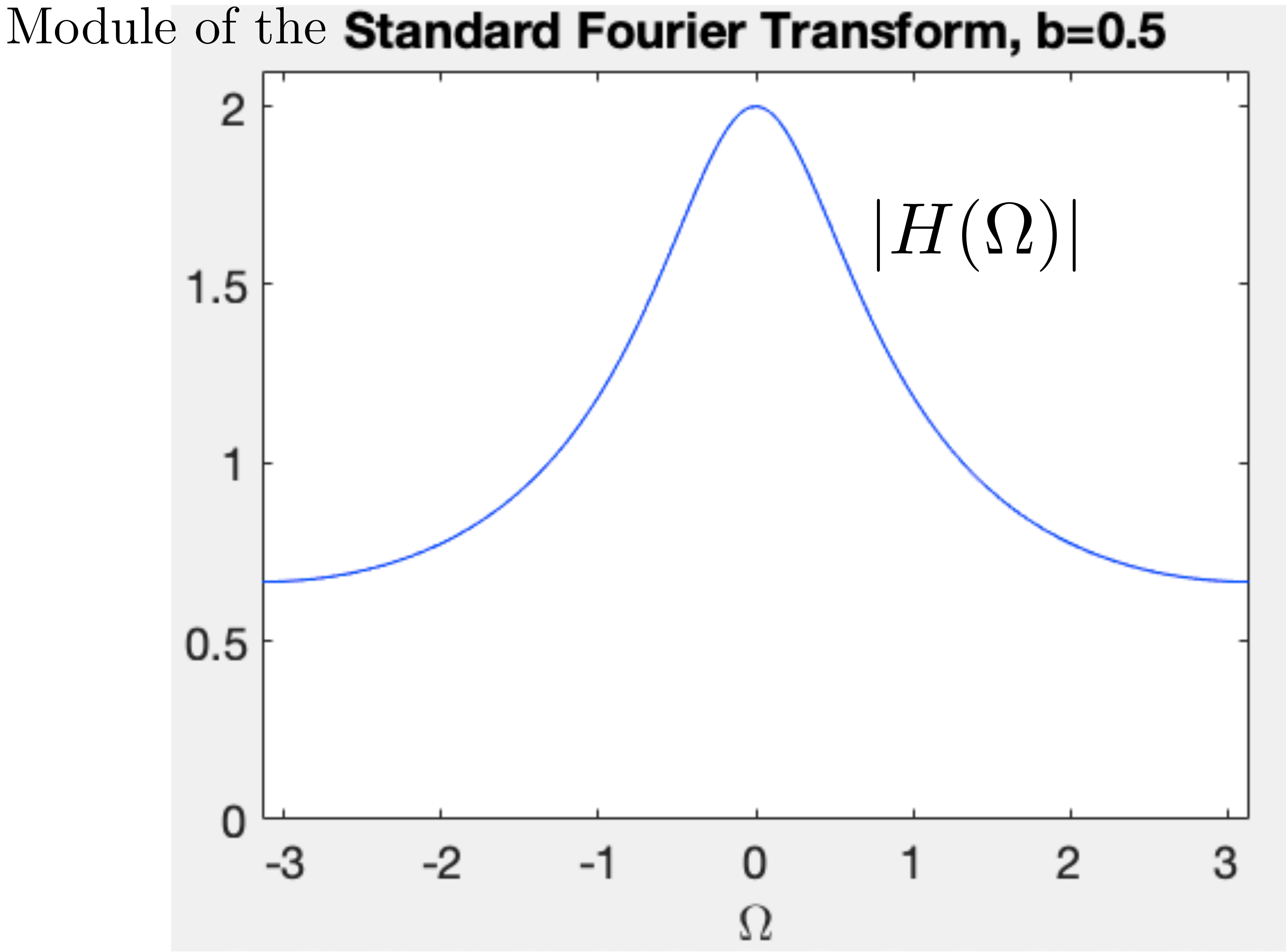

Low-pass...but not "very strong"
non very selective....

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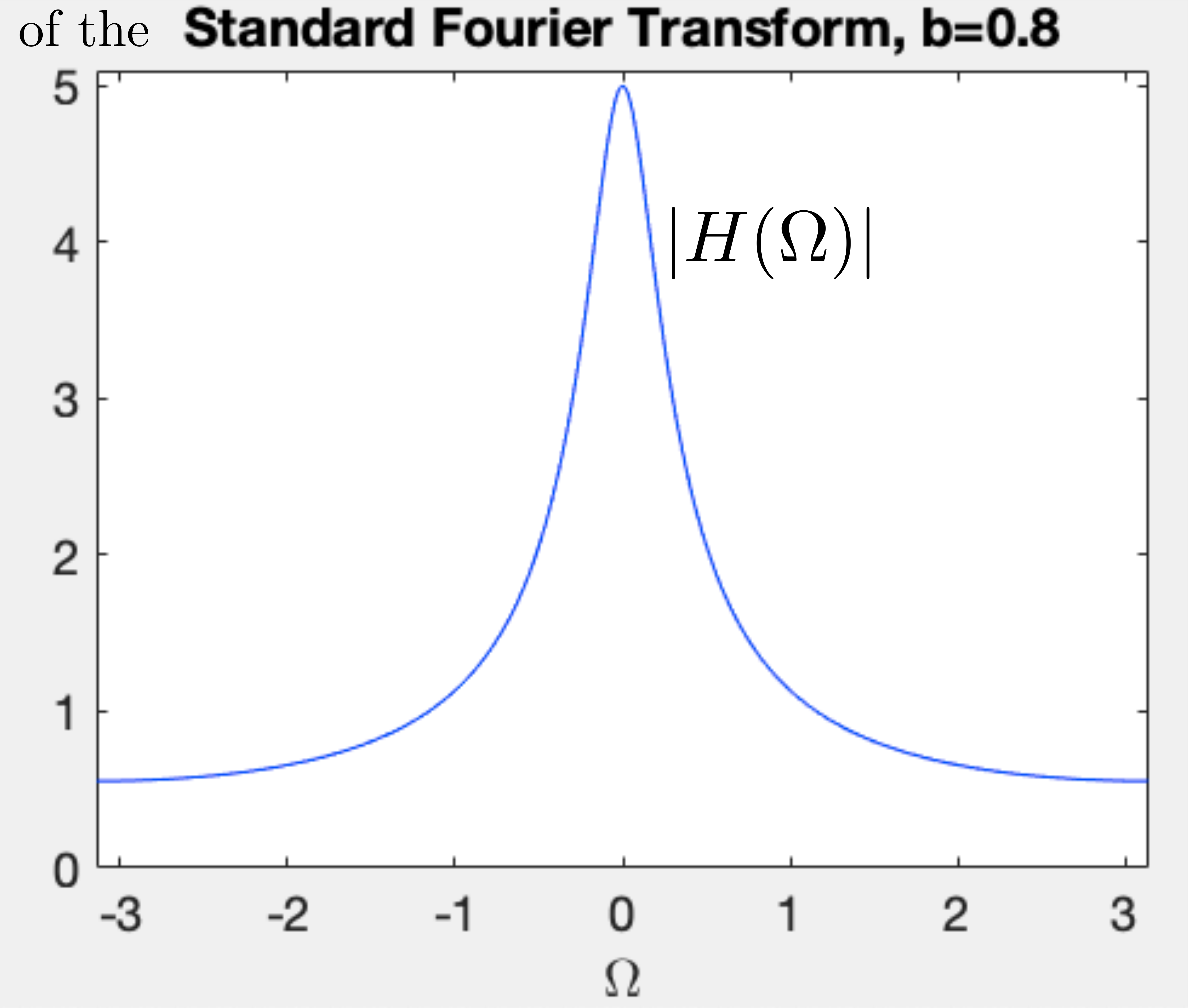
Module of the



Low-pass filter, more selective….

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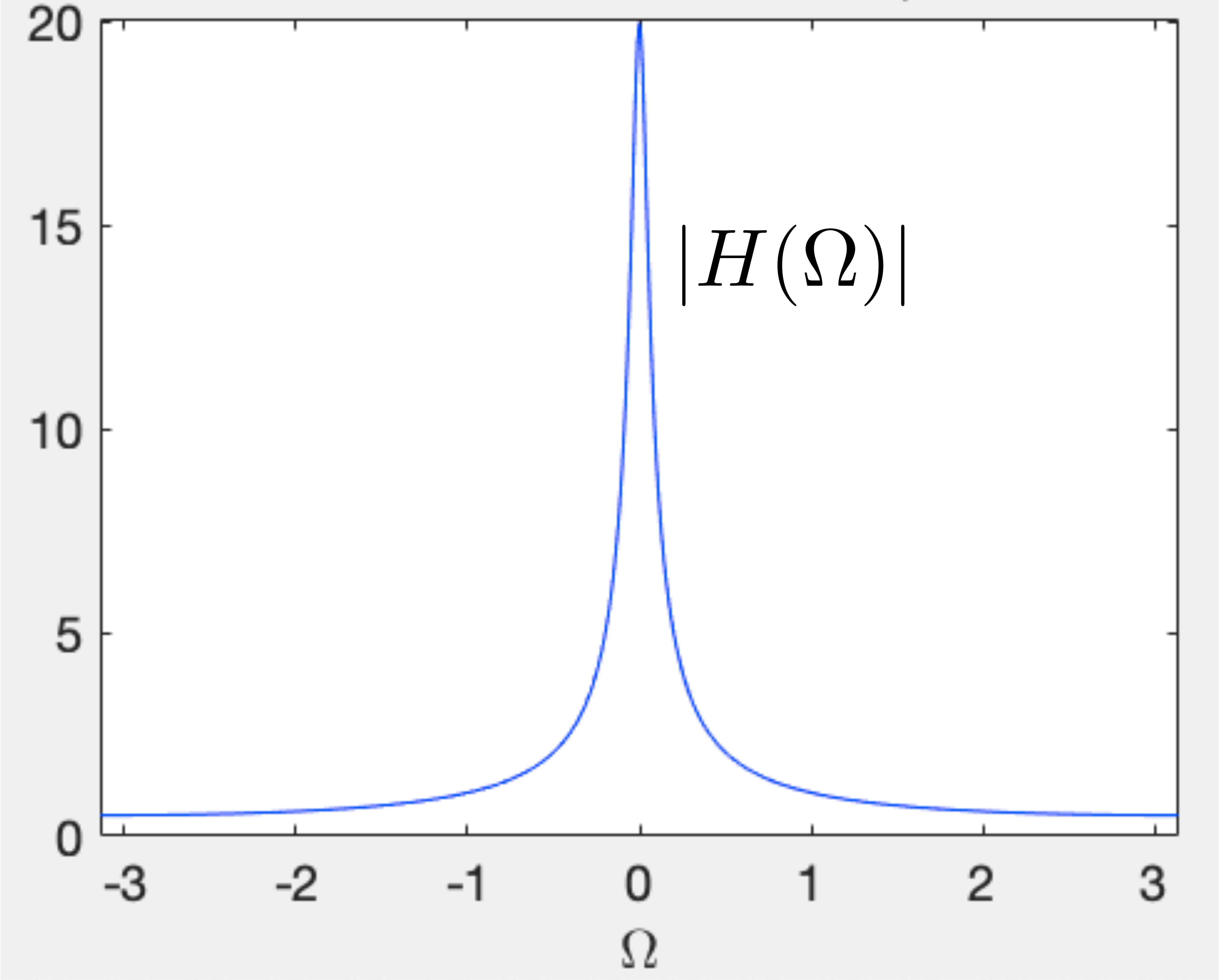
Module of the **Standard Fourier Transform, b=0.95**



Low-pass filter, very selective !

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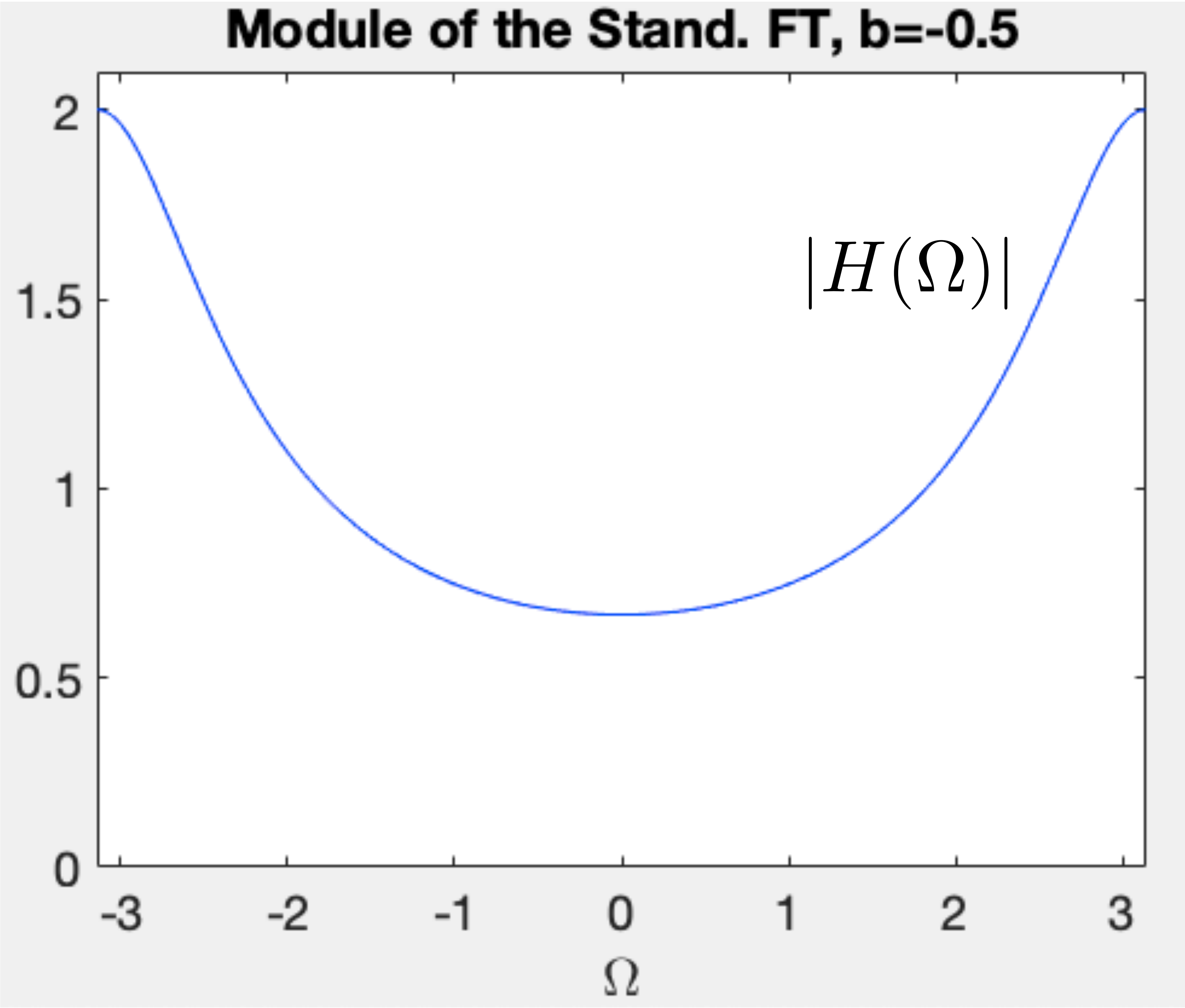

High-pass…but not "very strong"
non very selective….

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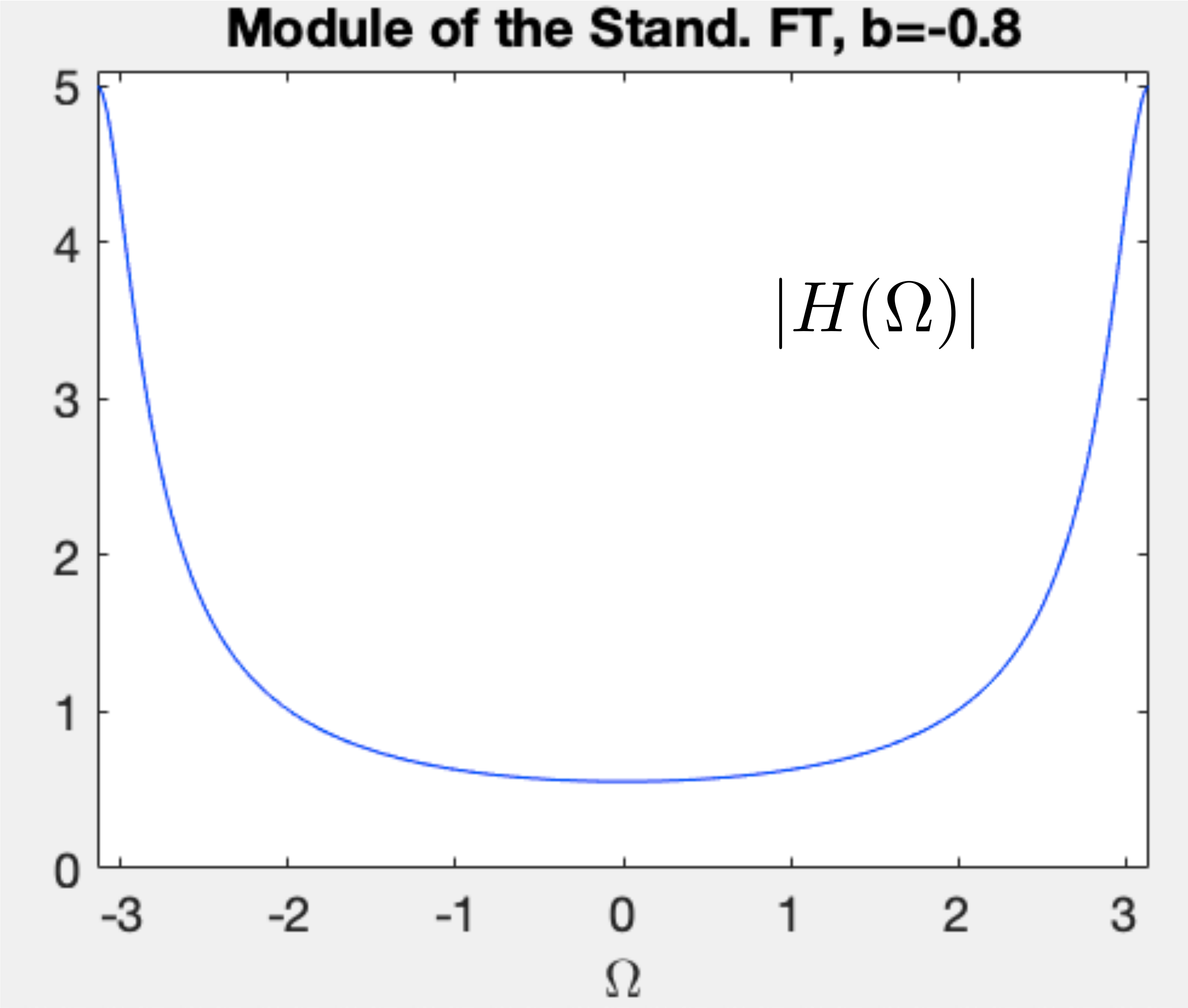

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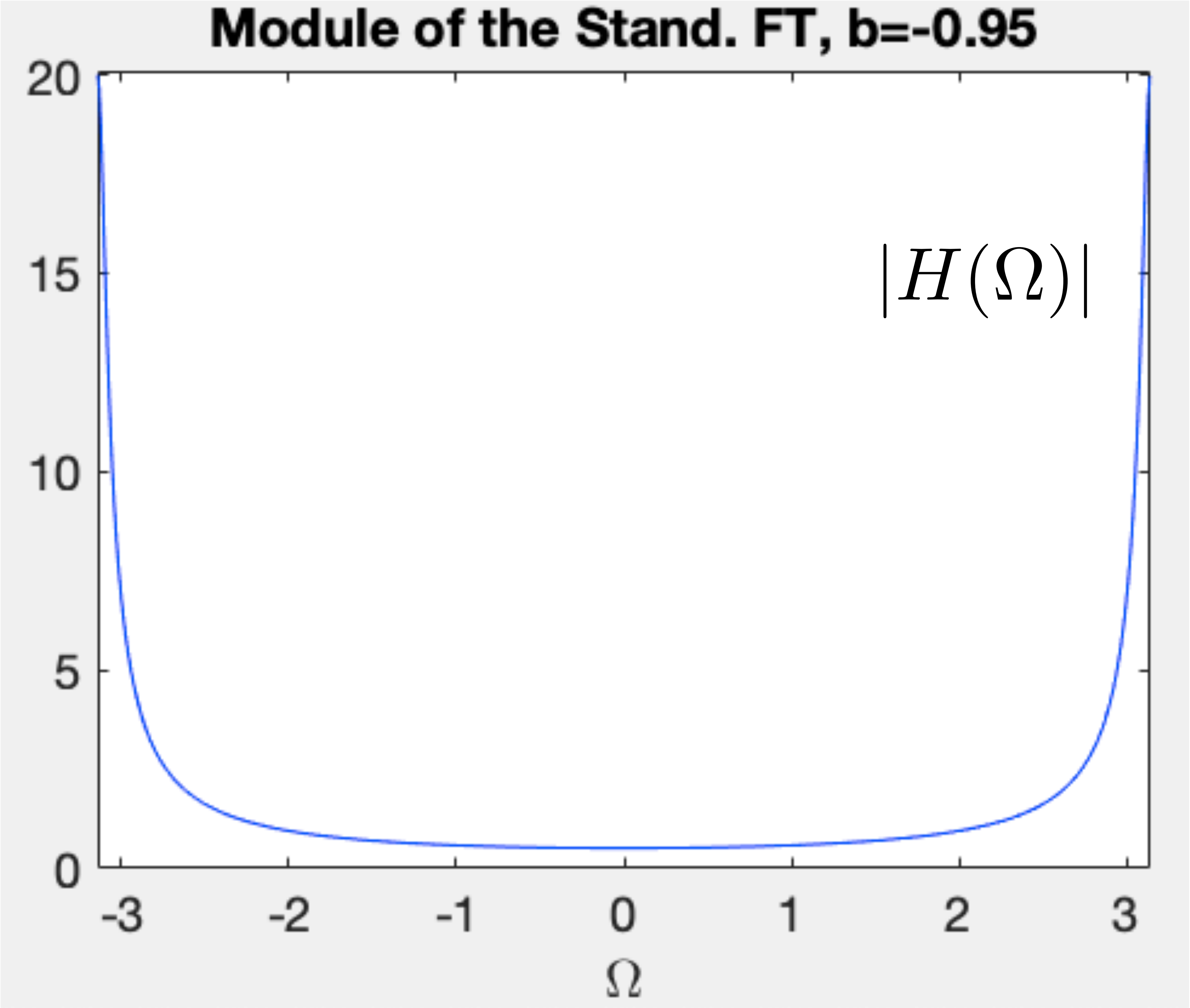

High-pass filter, very selective !

```
clear all
close all
clc

%%%% EN MATLAB TODOS LOS VECTORES EMPIEZAN EN 1
T=15; %%%% longitud total --- en realida es T-2
y(1)=0; %%% seria y[-1]
x(1)=0; %%% seria x[-1]
x(2)=1; %%% seria x[0]
x(3:T)=zeros(1,T-2);
%%%%%%%%%%%%%%%%%%%%%%%%%
b=-0.95;
for n=2:T %%%% n=2 es en realidad n=0;
    y(n)=b*y(n-1)+x(n);
end
plot([0:T-2],y(2:T),'o-')
%hold on
set(gca,'FontSize',20)
xlabel('n')
title(['h[n]=y[n] with b=',num2str(b)])
%axis([0 15 -1 1])
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Omega=-pi:0.01:pi;
Homega=exp(j*Omega)./(exp(j*Omega)-b);
figure
plot(Omega,abs(Homega),'b')
set(gca,'FontSize',20)
xlabel('\Omega')
title(['Module of the Stand. FT, b=',num2str(b)])
axis([-pi pi 0 max(abs(Homega))+0.1])
```

Questions?