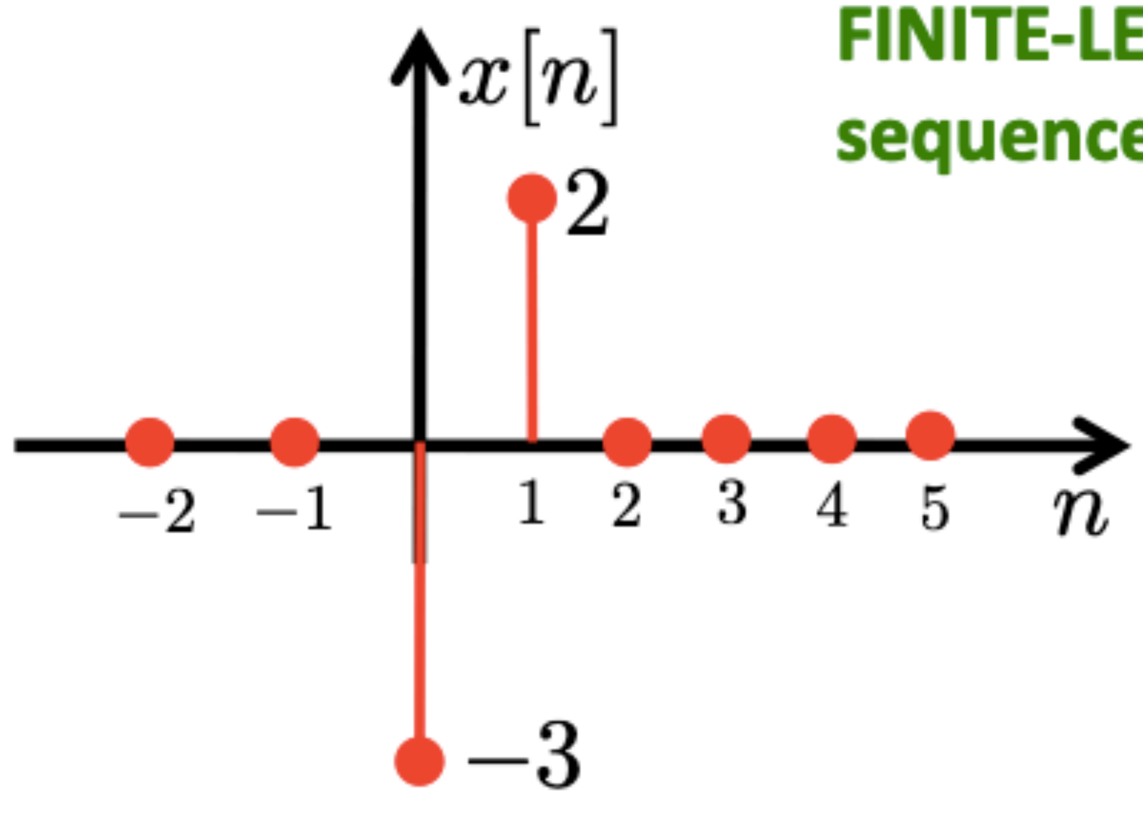
Extending the concept of left-sided and right-sided sequences/signals

First we recall some example already seen…

First recalling:

Example 10

$x[0] = -3, x[1] = 2,$
otherwise 0



FINITE-LENGTH sequence

Easy to express with deltas

$$x[n] = -3\delta[n] + 2\delta[n-1]$$

It could be considered: right-sided (and it is finite-length)

Example 10

$$x[n] = -3\delta[n] + 2\delta[n-1]$$

Using the result of example 7:

$$x[n] = \delta[n - n_0] \longrightarrow X(z) = z^{-n_0}$$

$$X(z) = -3 + 2z^{-1}$$

$$X(z) = \frac{-3z + 2}{z}$$

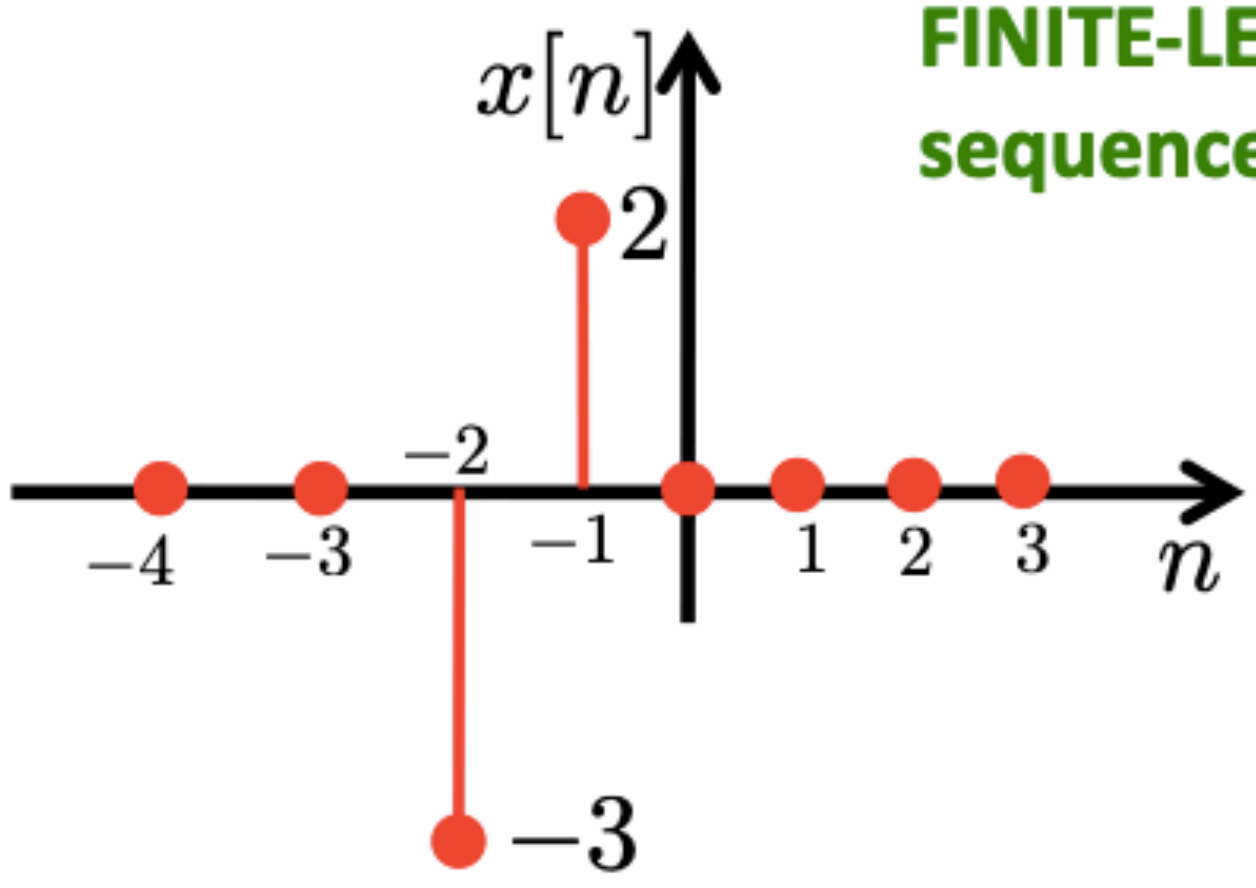
ROC: all the complex plane except z=0! (i.e., r=0)

First recalling:

Example 11

$x[-2] = -3, \quad x[-1] = 2$

otherwise 0



FINITE-LENGTH sequence

Easy to express with deltas

$$x[n] = -3\delta[n+2] + 2\delta[n+1]$$

It could be considered: left-sided (and it is finite-length)

Example 11

$$x[n] = -3\delta[n+2] + 2\delta[n+1]$$

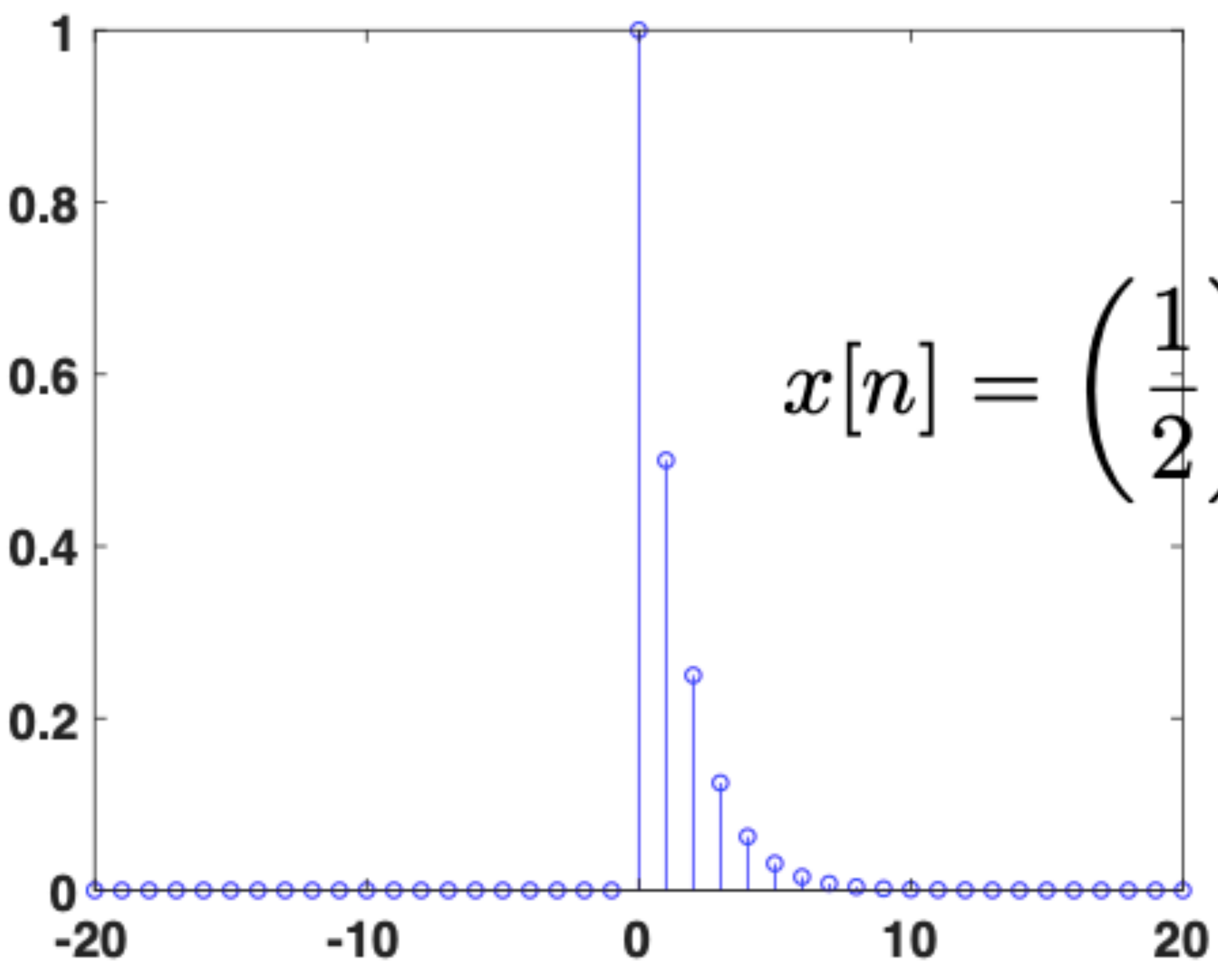
$$X(z) = -3z^2 + 2z$$

ROC: all the complex plane except |z|=r=Infinity!

First recalling:

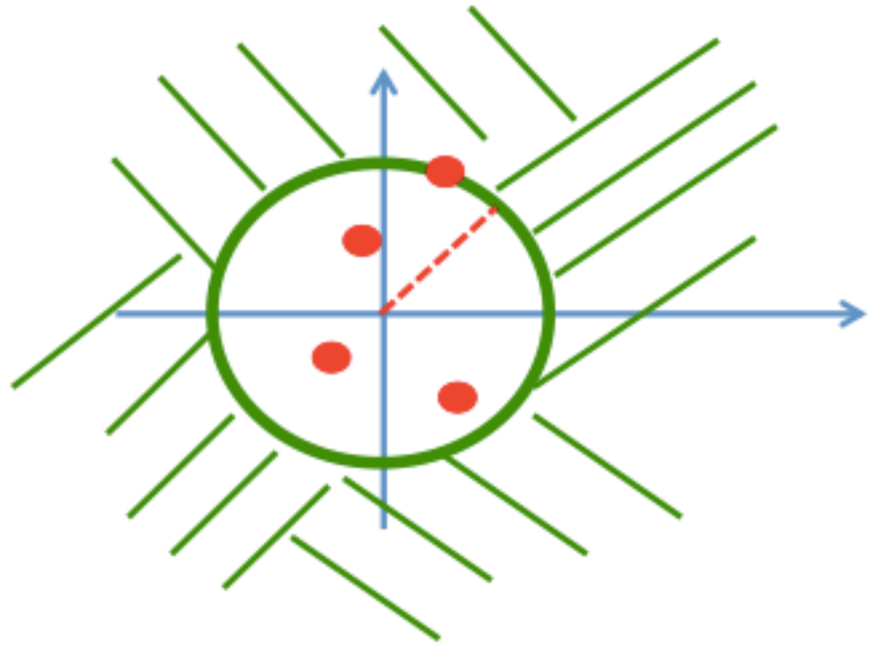
Right-sided sequence-signal

Other examples of right-sided sequences, x[n]=(1/2)^n u[n]



$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$

Poles in red dots !



ROC “outside” !

Determined by the pole with biggest module.

$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$

X(z) and ROC:

$$X(z) = \frac{z}{z - \frac{1}{2}}$$

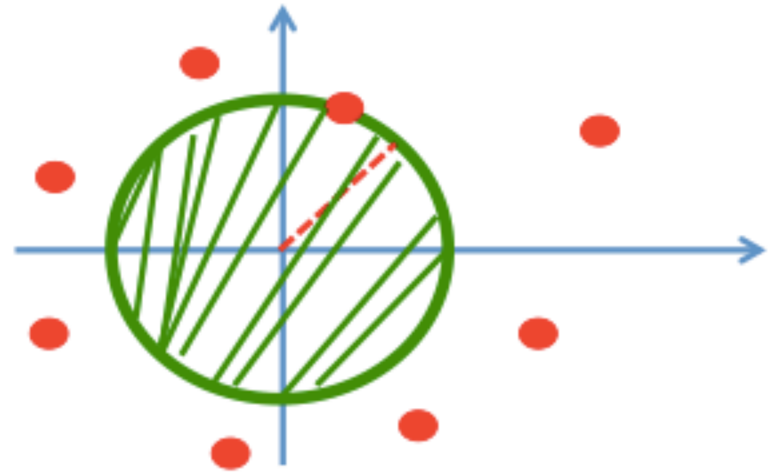
$$|z| > \frac{1}{2}$$

$$r > \frac{1}{2}$$

it is a right-sided (and it has an infinite-length)

First recalling:

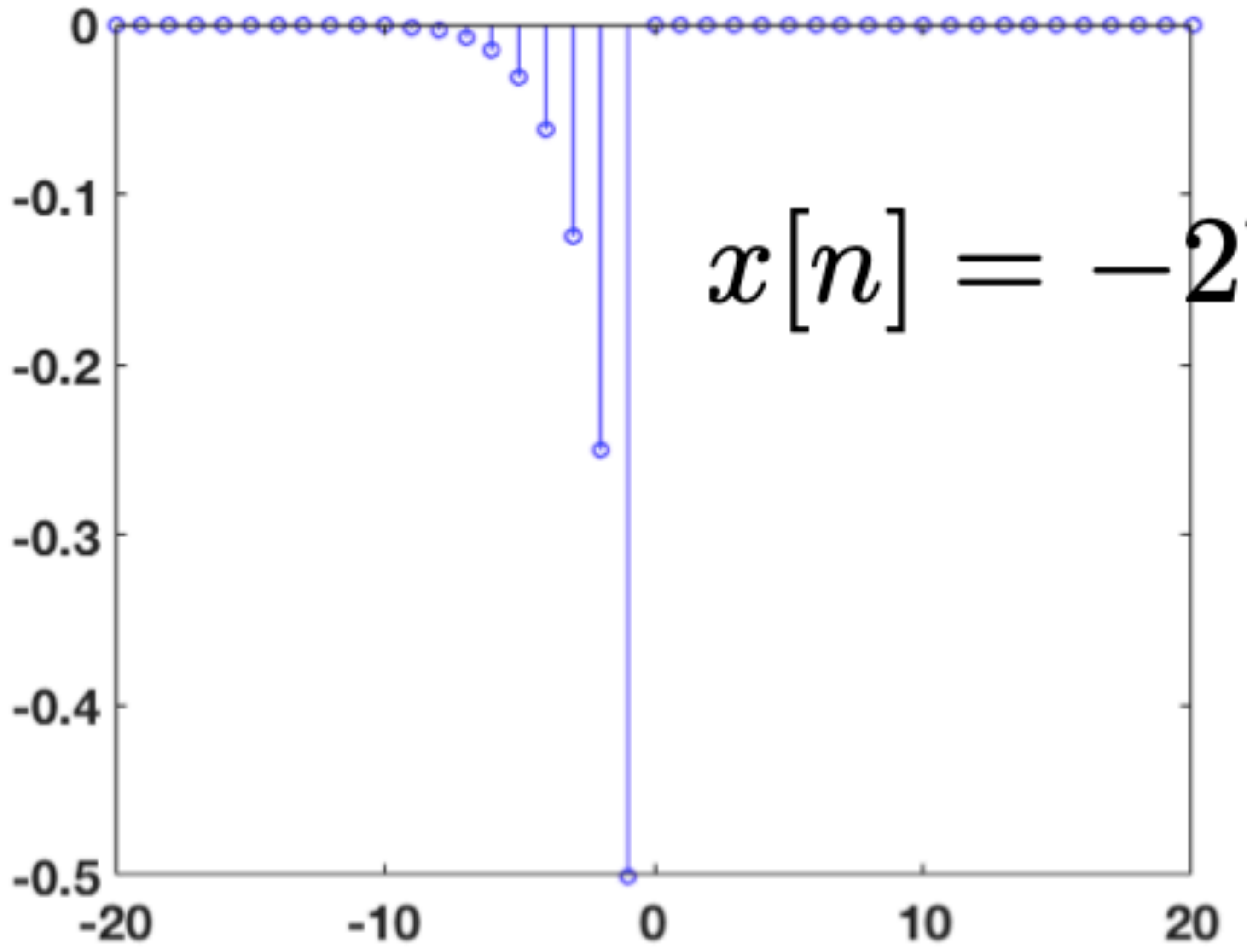
Poles in red dots !



LEFT-sided sequence-signal

Another example of right-sided sequences, x[n]=-2^(n) u[-1-n]

ROC "inside" !
Determined by the pole with smallest module.



$$x[n] = -2^n u[-n-1]$$

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X(z) and ROC:

$$|z| < 2$$

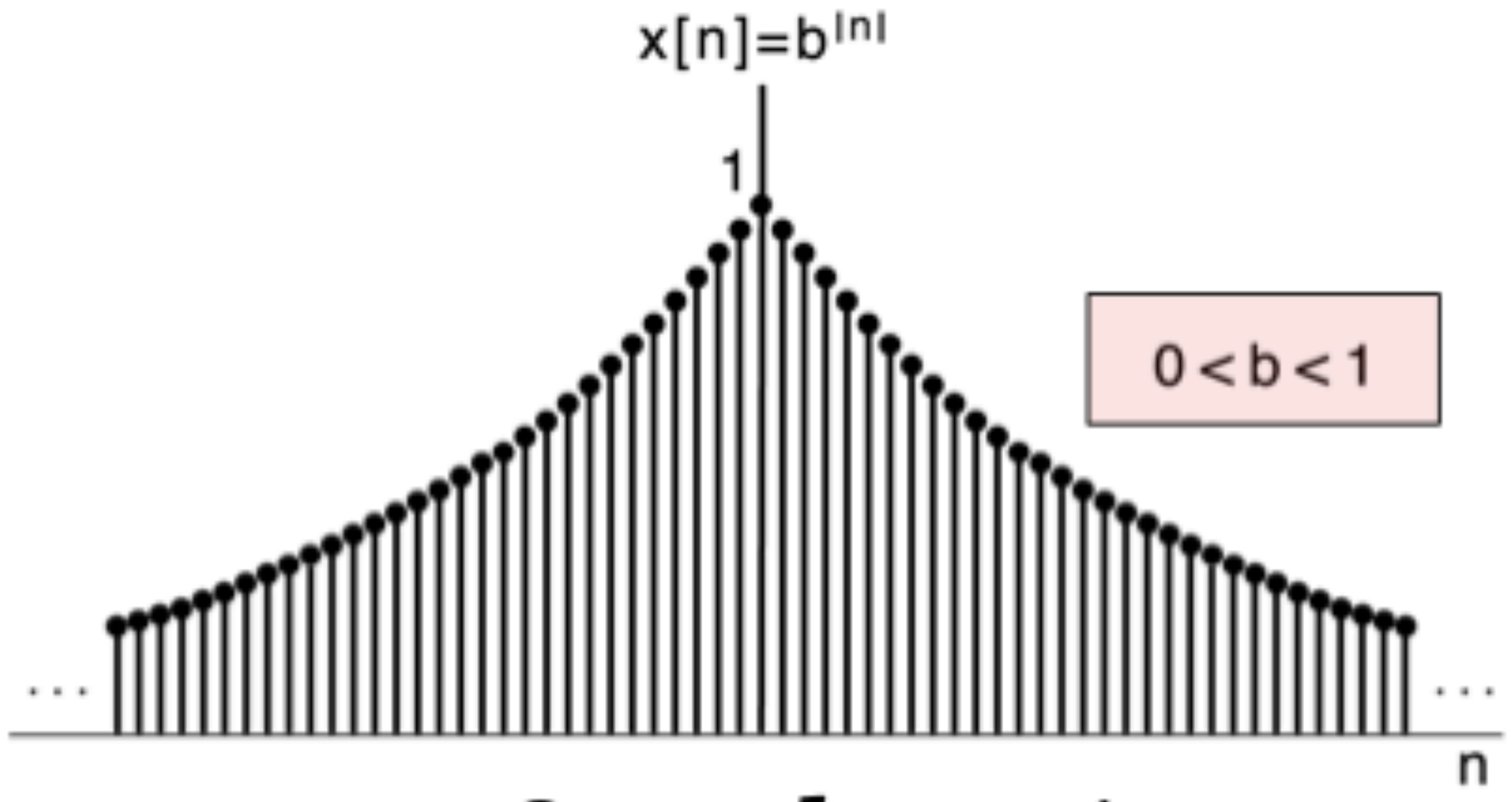
$$X(z) = \frac{z}{z-2}$$

it is a left-sided (and it has an infinite-length)

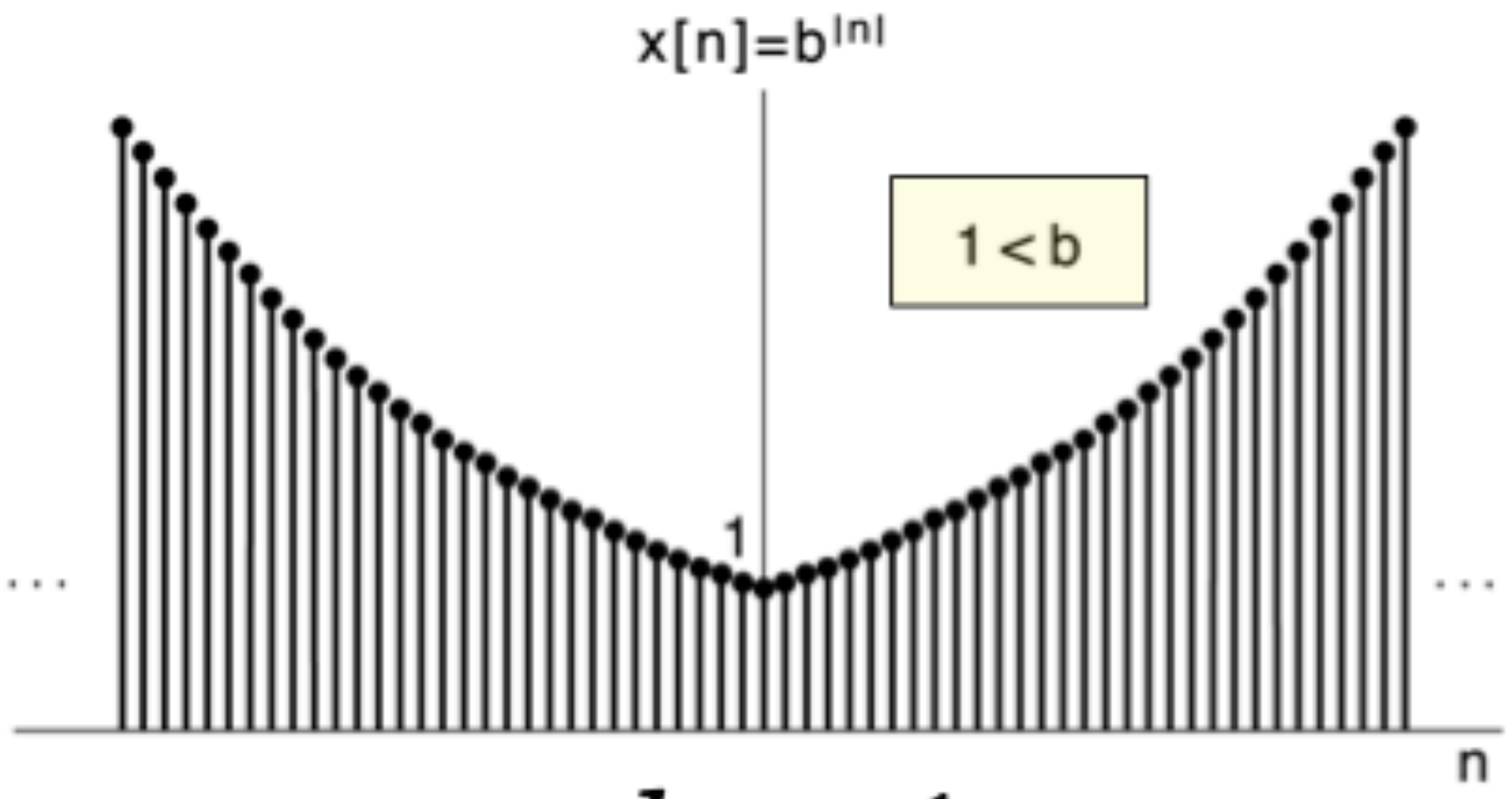
Both-sided or Two-sided, for sure (100%):

Example 3

$$x[n] = b^{|n|}, \quad b > 0$$



$0 < b < 1$



$b > 1$

and the ZT exists only in one case, and with “donut” ROC:

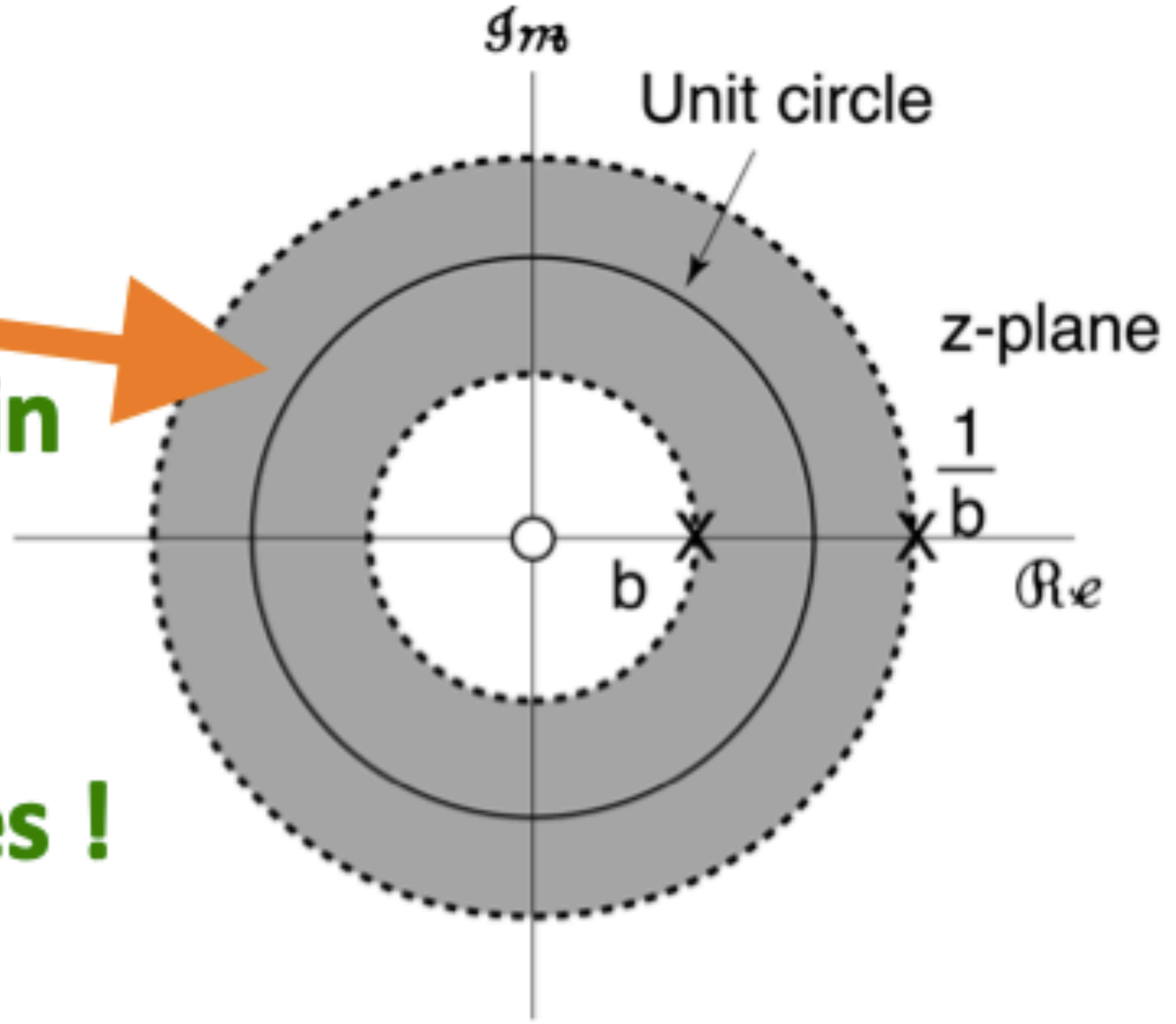
Solution for b<1:

$$X(z) = \frac{1}{1 - bz^{-1}} + \frac{-1}{1 - b^{-1}z^{-1}} \quad , \quad b < |z| < \frac{1}{b}$$

...it can be also written with a unique fraction....

✓The FT always exists! The unit circle is always within The ROC !!

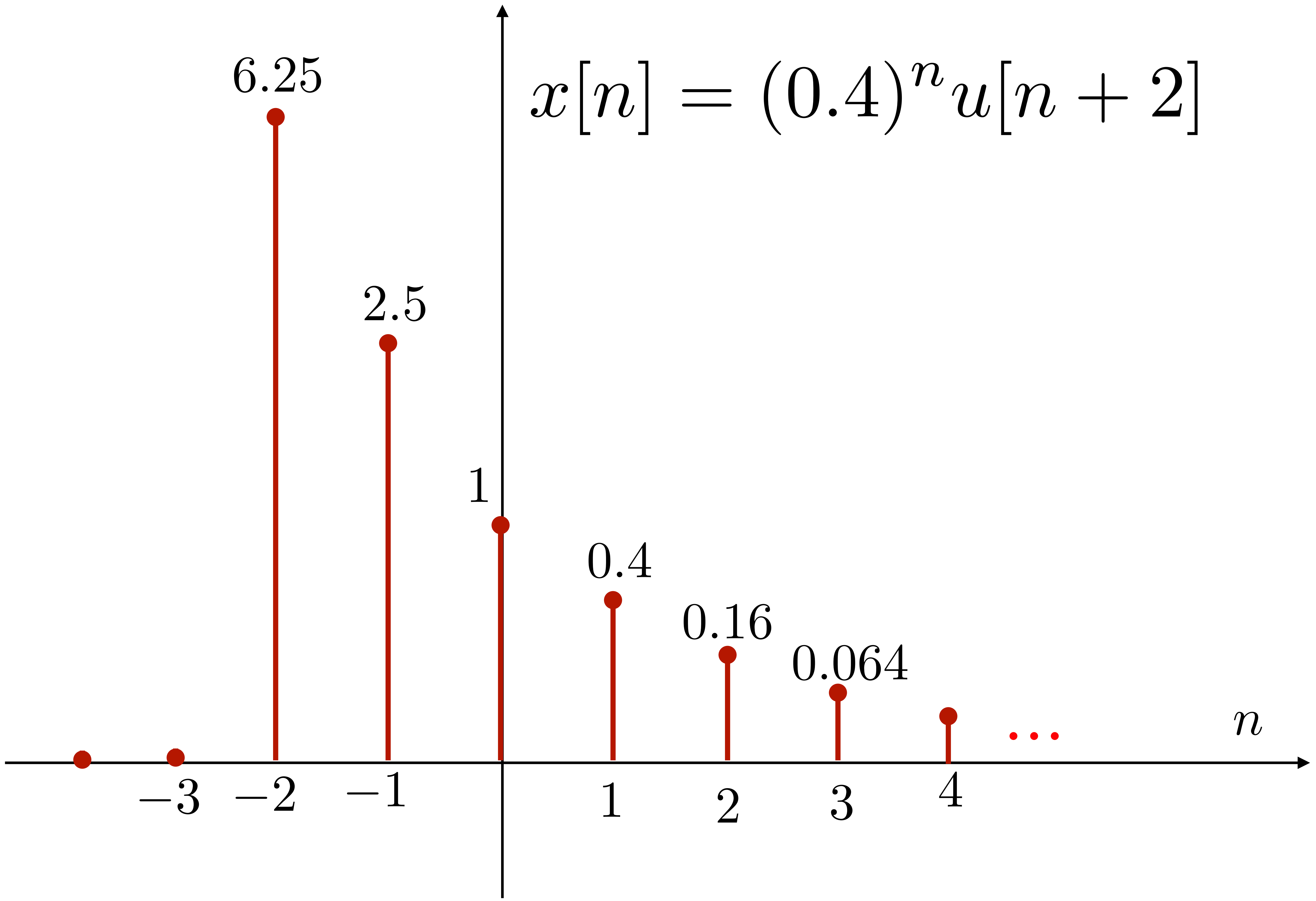
✓ b and 1/b are the two poles !
✓ we have a “DONUT” ROC !



Now we can “mix”/combine some previous cases…

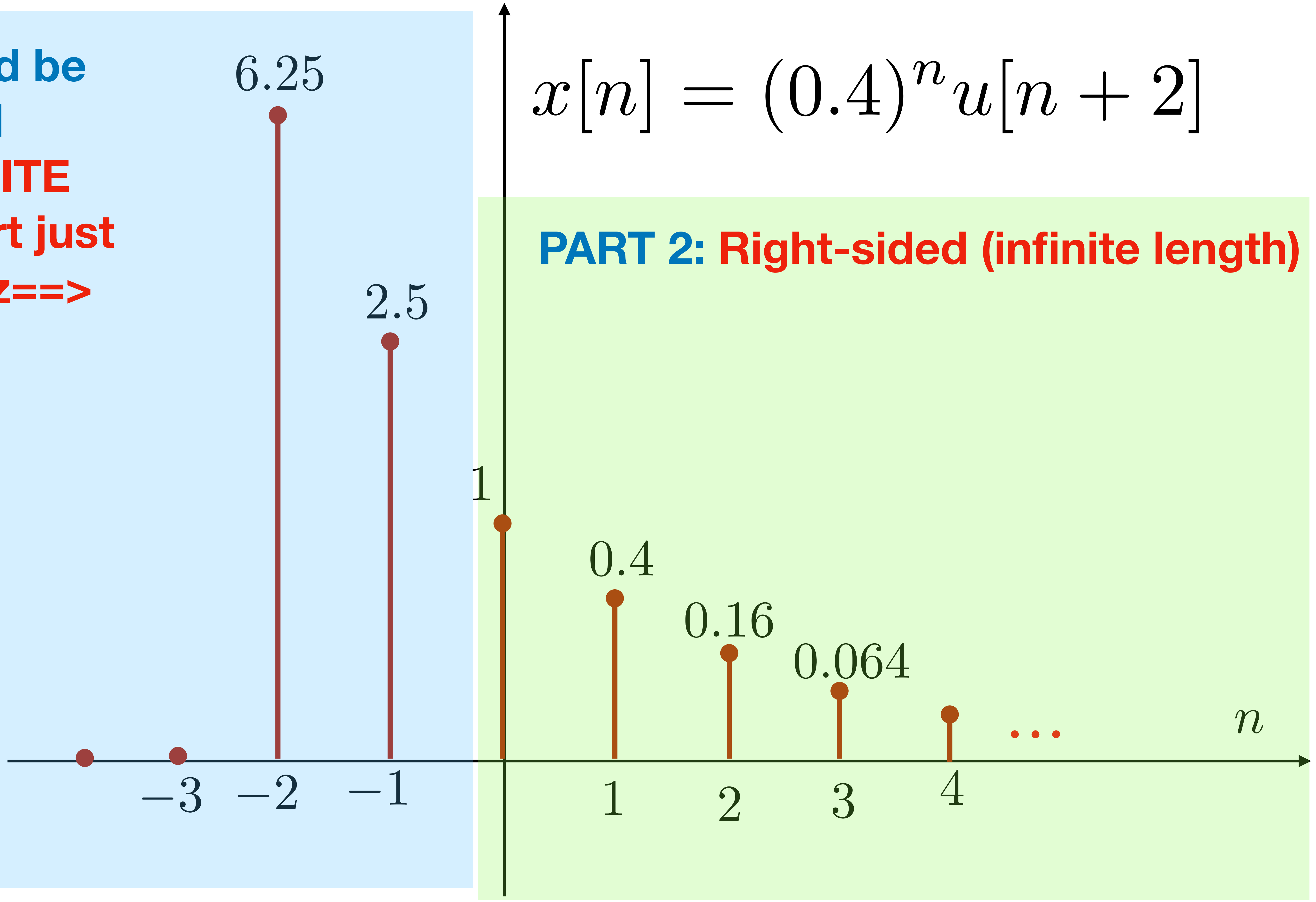
FIRST “MIX” EXAMPLE

Consider now this example:



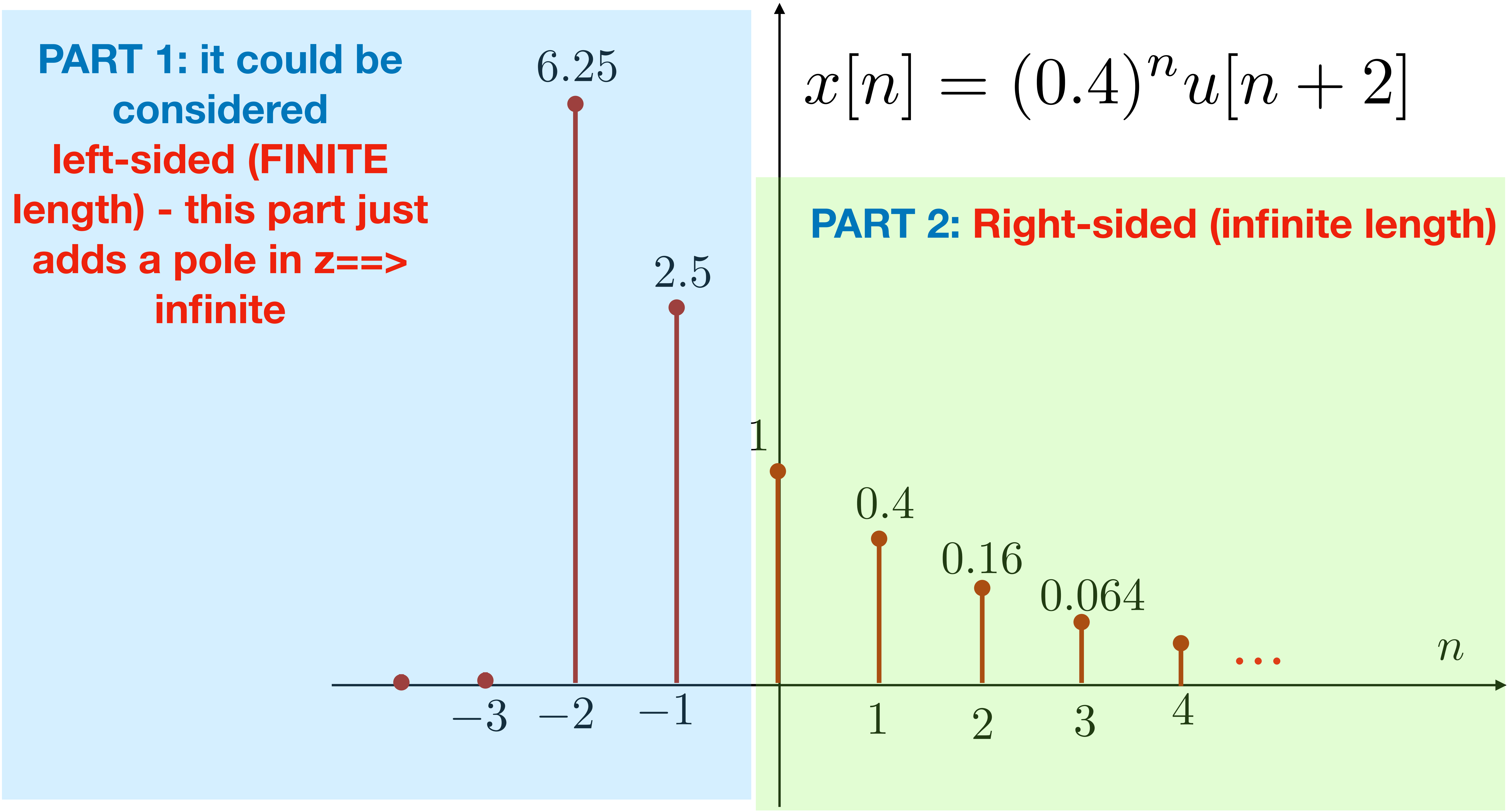
We can see this signal as formed by two parts:

PART 1: it could be considered left-sided (FINITE length) - this part just adds a pole in z==> infinite



The samples at n=0 can belong to both "parts" - no problem

We can see this signal as formed by two parts:



This signal is “more” right-sided, since has infinite length in the right-side… So many people say: right-sided signal

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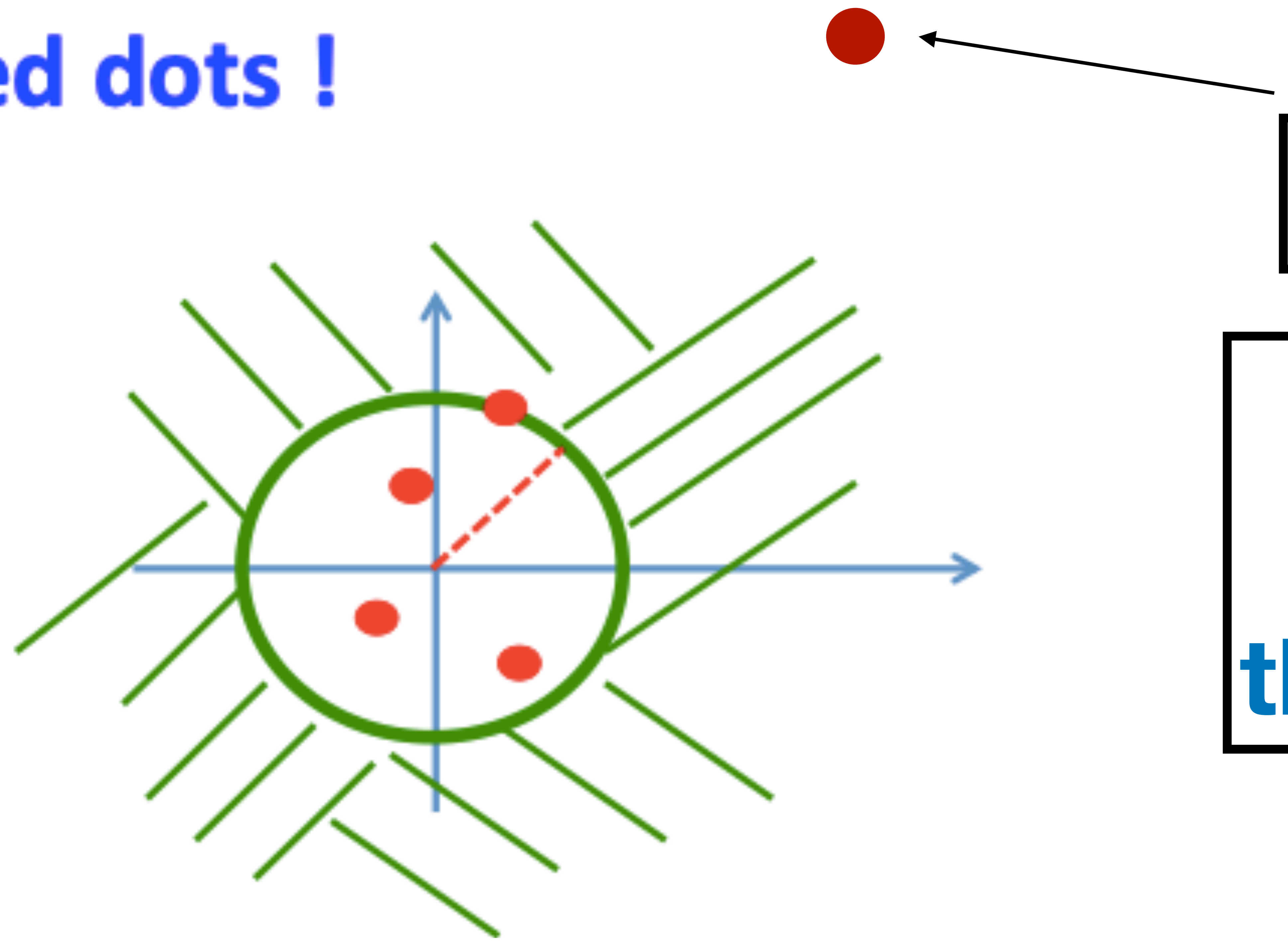
Poles in red dots !



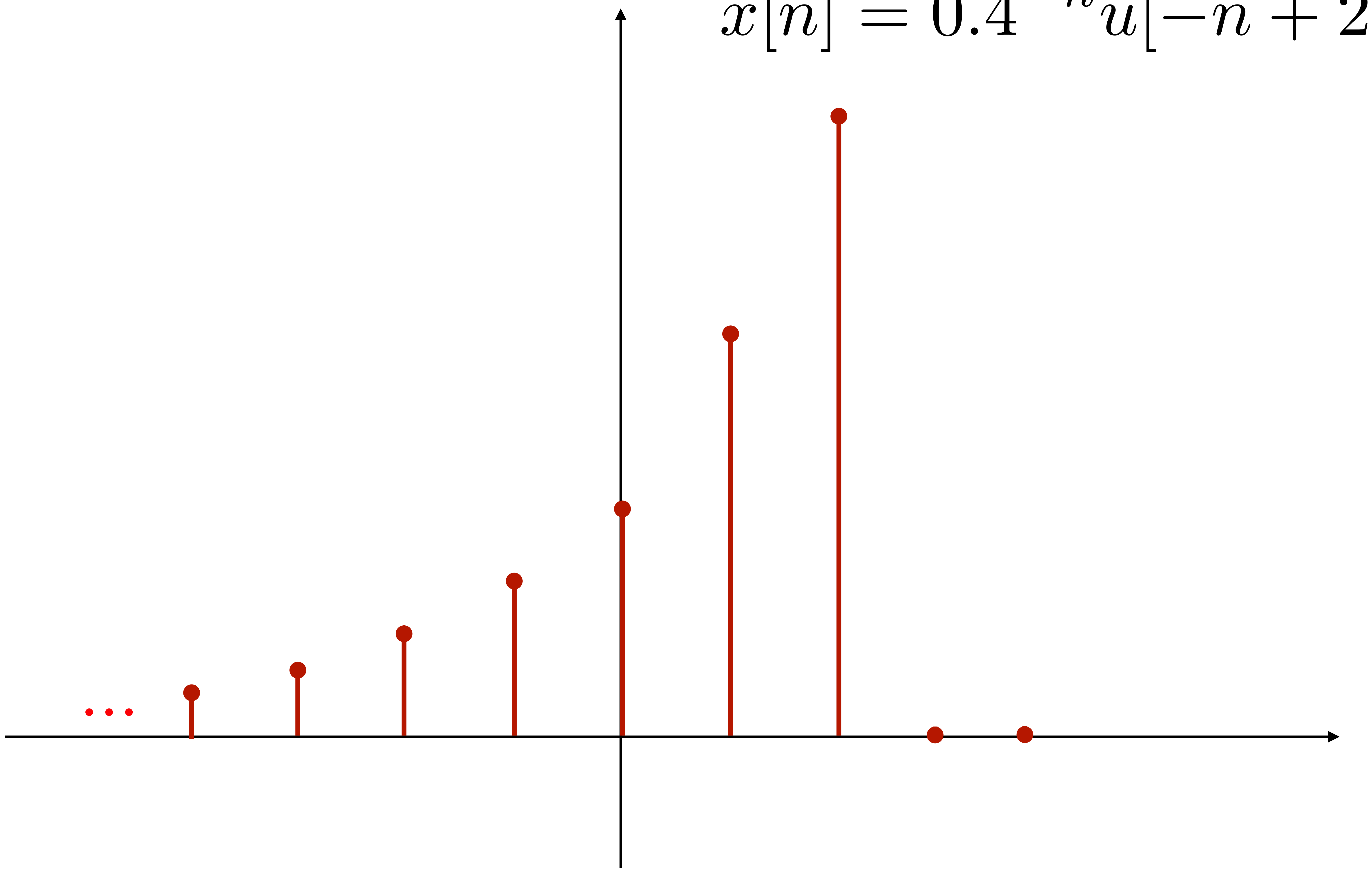
There is a pole in infinite

It is like a “donut” ROC, but where the bigger radius is infinite

SECOND “MIX” EXAMPLE

Consider now this example:

$$x[n] = 0.4^{-n}u[-n+2]$$



Consider now this example:

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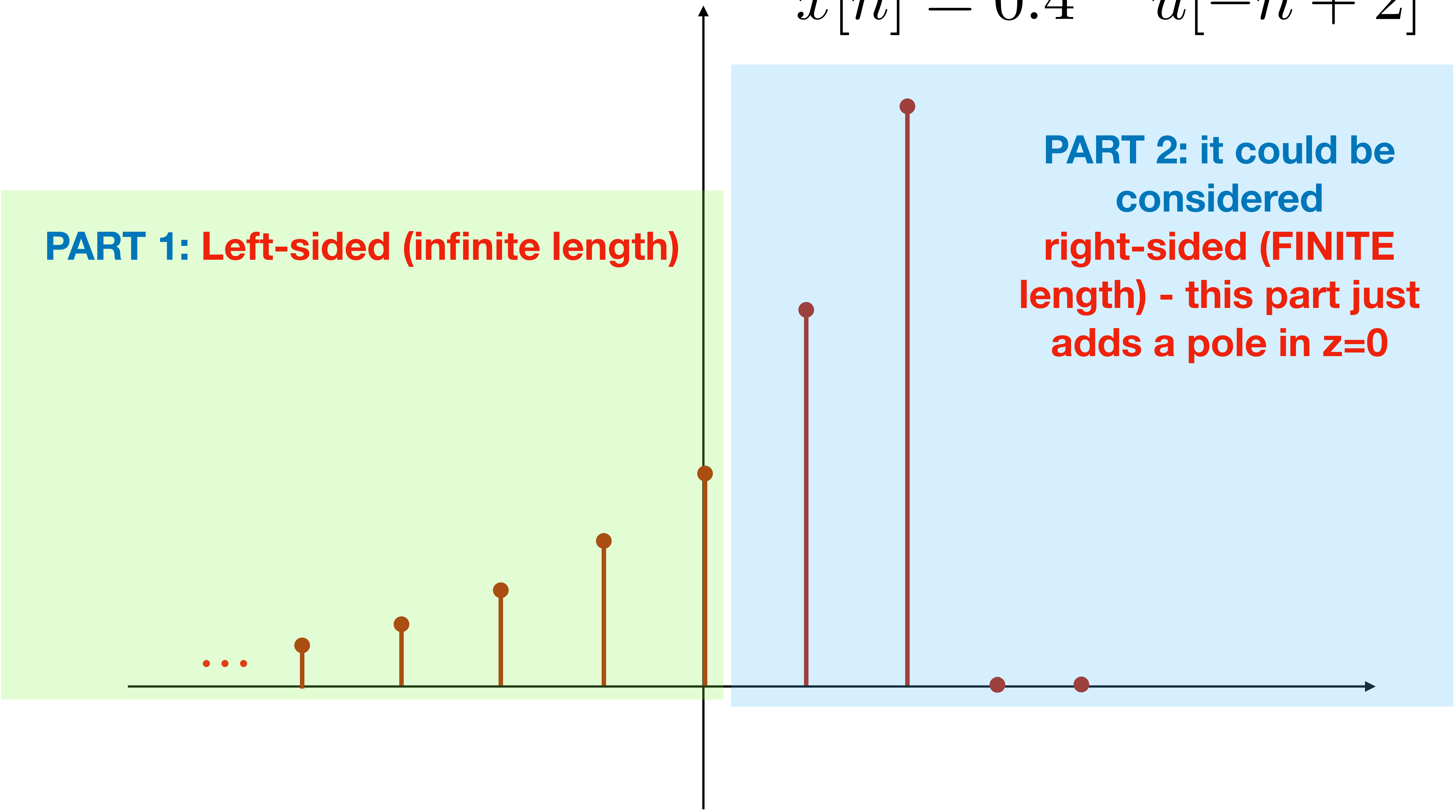
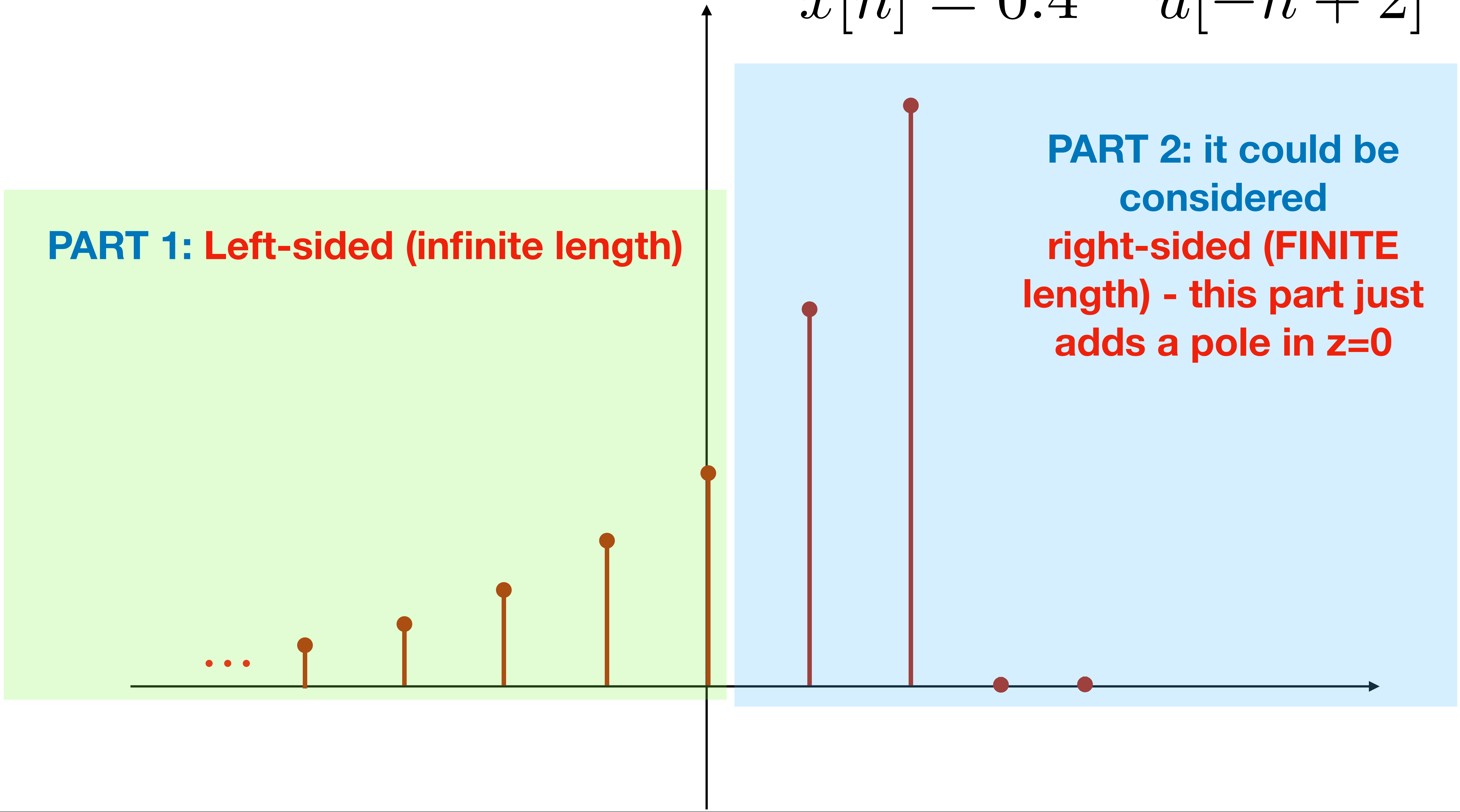


The samples at n=0 can belong to both "parts" - no problem

Consider now this example:

$$x[n] = 0.4^{-n} u[-n+2]$$



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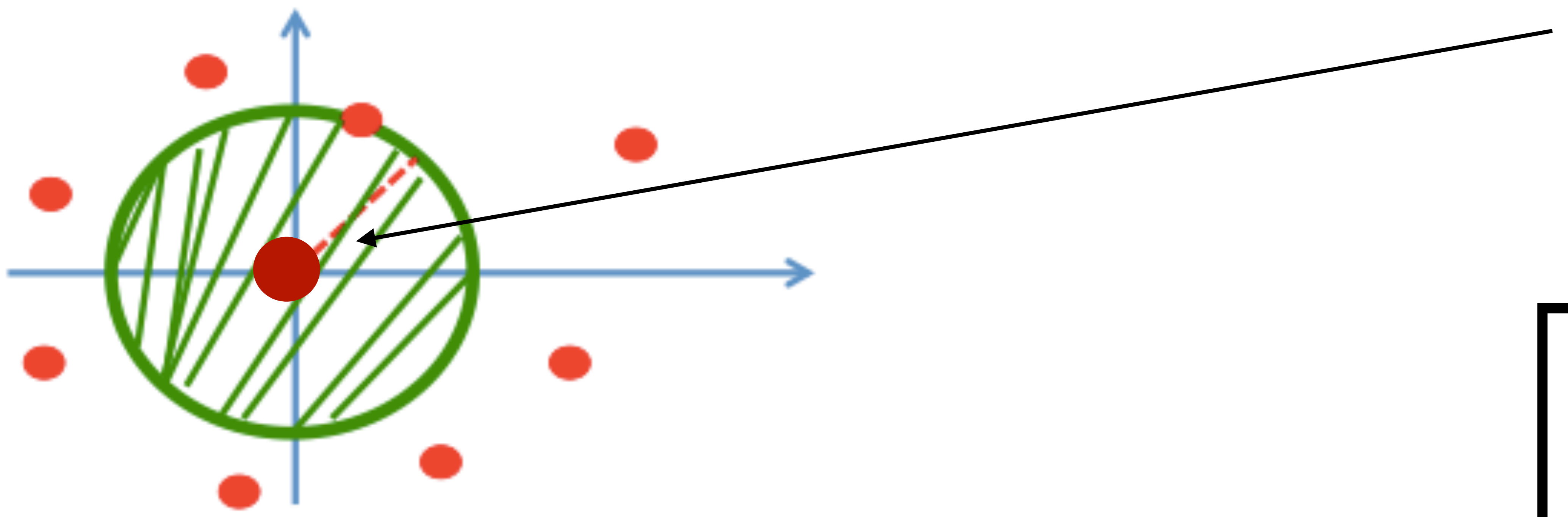

There is a pole at z=0

It is like a "donut" ROC, but where the smaller radius is zero