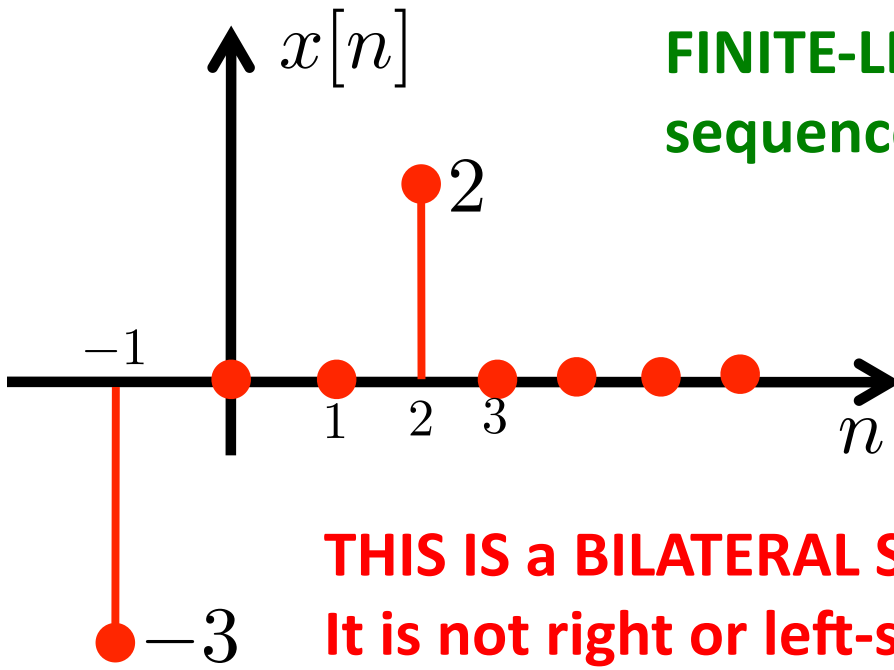
TOPIC 5
ZETA TRANSFORM
PART 4

RECALL: Example 13

$x[-1] = -3, \quad x[2] = 2$
otherwise 0



FINITE-LENGTH sequence

THIS IS a BILATERAL SEQUENCE !!
It is not right or left-sided, it is both….

Easy to express with deltas

$$x[n] = -3\delta[n+1] + 2\delta[n-2]$$

RECALL: Example 13

$$x[n] = -3\delta[n+1] + 2\delta[n-2]$$

$$X(z) = -3z + 2z^{-2}$$

$$X(z) = \frac{-3z^3 + 2}{z^2}$$

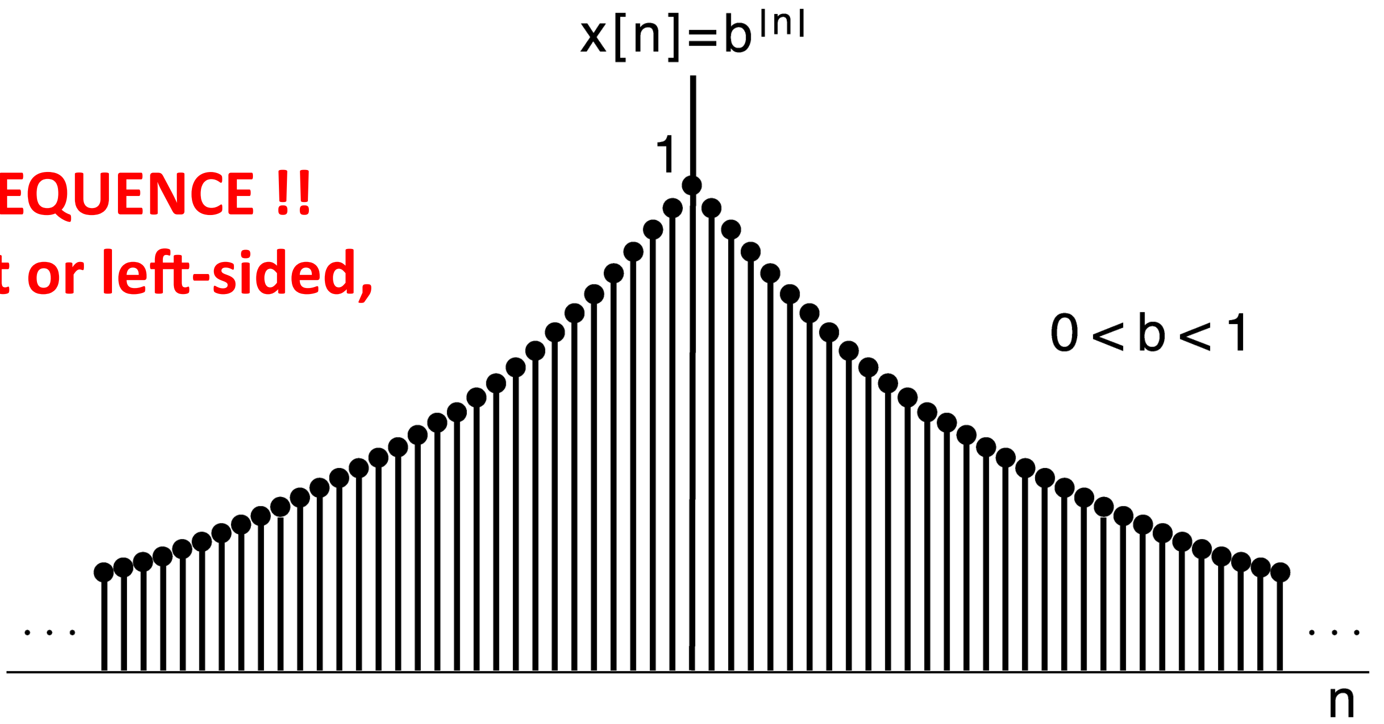
Note that degree N(z) > degree D(z)

ROC: all the complex plane except z=r=0 (pole multiple of degree 2) and |z|=r=Infinity (it is another pole)

Recalling also the signal…

$$x[n] = b^{|n|}, \quad 0 < b < 1$$

THIS IS a
BILATERAL SEQUENCE !!
It is not right or left-sided,
it is both….

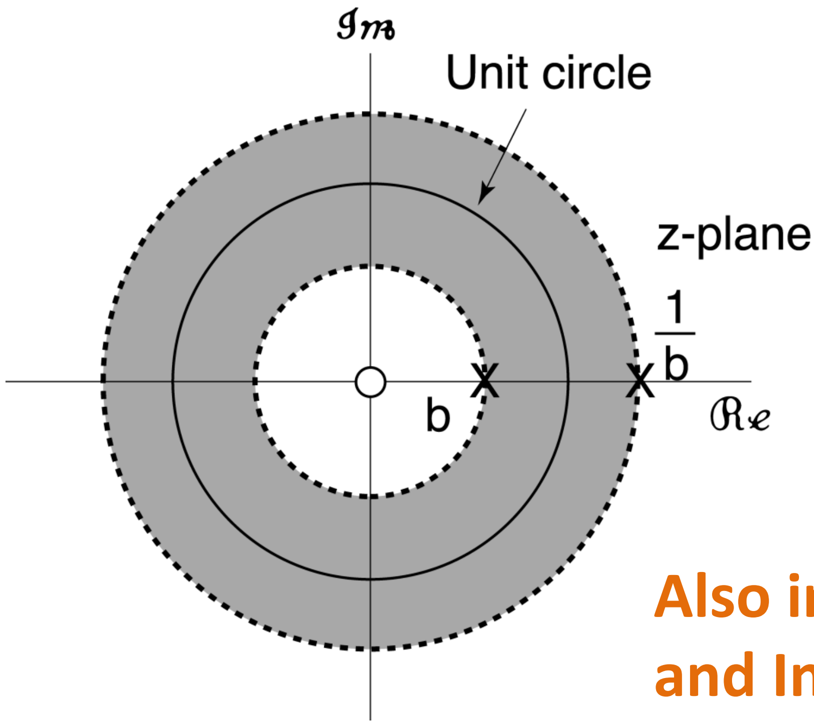


…the ZT and ROC was…

Solution for 0<b<1:

$$X(z) = \frac{1}{1 - bz^{-1}} + \frac{-1}{1 - b^{-1}z^{-1}} \quad , \quad b < |z| < \frac{1}{b}$$

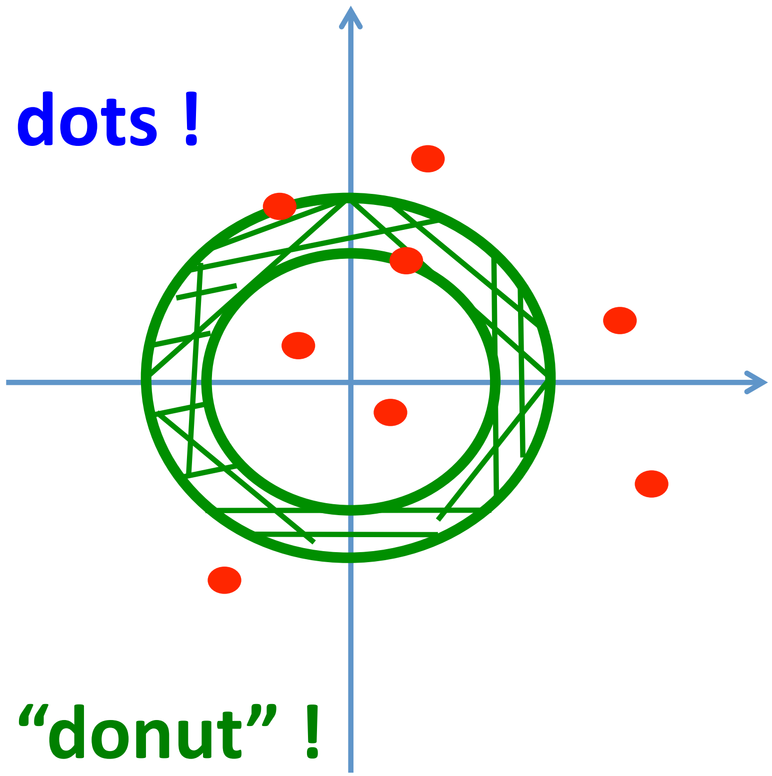
ROC:



Also in this case 0
and Infinity are not included !!

ROCs for
BILATERAL SEQUENCES- SIGNALS

Poles in red dots !



ROC like a "donut" !

Recall: the ROC cannot contain a pole.

SUMMARY

RIGHT-SIDED signal → ROC “outside”

Infinity is contained;

zero is NOT contained

LEFT-SIDED signal → ROC “inside”

Infinity is NOT contained;

zero is contained

BILATERAL signal → ROC “donut”

Infinity is NOT contained;

zero is NOT contained

SUMMARY: if also is a FINITE-LENGTH signal

ROC is all the complex plane but:

RIGHT-SIDED signal (and finite-length) → Infinity is contained; zero is NOT contained

LEFT-SIDED signal (and finite-length) → Infinity is NOT contained; zero is contained

BILATERAL signal (and finite-length) → Infinity is NOT contained; zero is NOT contained

SUMMARY:
if it is a FINITE-LENGTH signal

TYPE of ZT – just examples:

RIGHT-SIDED signal (and finite-length) → $$X(z) = \frac{z^2 + 3z - 1}{z^2}$$

Degree of numerator < or = degree of denominator

LEFT-SIDED signal (and finite-length) → $$X(z) = z^4 - z^2 + 3$$

Only numerator

BILATERAL signal (and finite-length) → $$X(z) = \frac{z^4 + 3z - 1}{z^2}$$

Degree of numerator > degree of denominator

MORE EXAMPLES

Example 14 - problem

Given the following Zeta Transform:

$$X(z) = \frac{3z^2 - \frac{5}{6}z}{(z - \frac{1}{4})(z - \frac{1}{3})}$$

Is there only one signal x[n] corresponding to this X(z)?

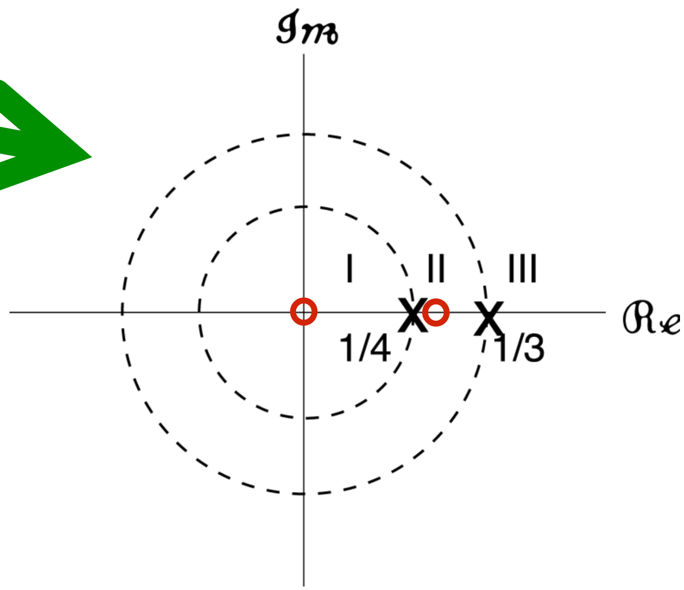
It can be more than 1, since we have not the ROC !!!

“Simple fractions” decomposition

In the next part – slides,
we will study how we will come back in time
with the so-called “Simple fractions” decomposition;
We will use quickly in this example.

Diagram of poles and zeros:

$$X(z) = \frac{3z^2 - \frac{5}{6}z}{(z-\frac{1}{4})(z-\frac{1}{3})}$$



Example 14 - problem

$$X(z) = \frac{3z^2 - \frac{5}{6}z}{(z - \frac{1}{4})(z - \frac{1}{3})} = \frac{3 - \frac{5}{6}z^{-1}}{(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{3}z^{-1})} = \frac{A}{1 - \frac{1}{4}z^{-1}} + \frac{B}{1 - \frac{1}{3}z^{-1}}$$

We obtain the values: A = 1, B = 2

$$X(z) = \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{2}{1-\frac{1}{3}z^{-1}}$$

$$\updownarrow \qquad \updownarrow \qquad \updownarrow$$

$$x[n] = x_1[n] + x_2[n]$$

Example 14 - problem

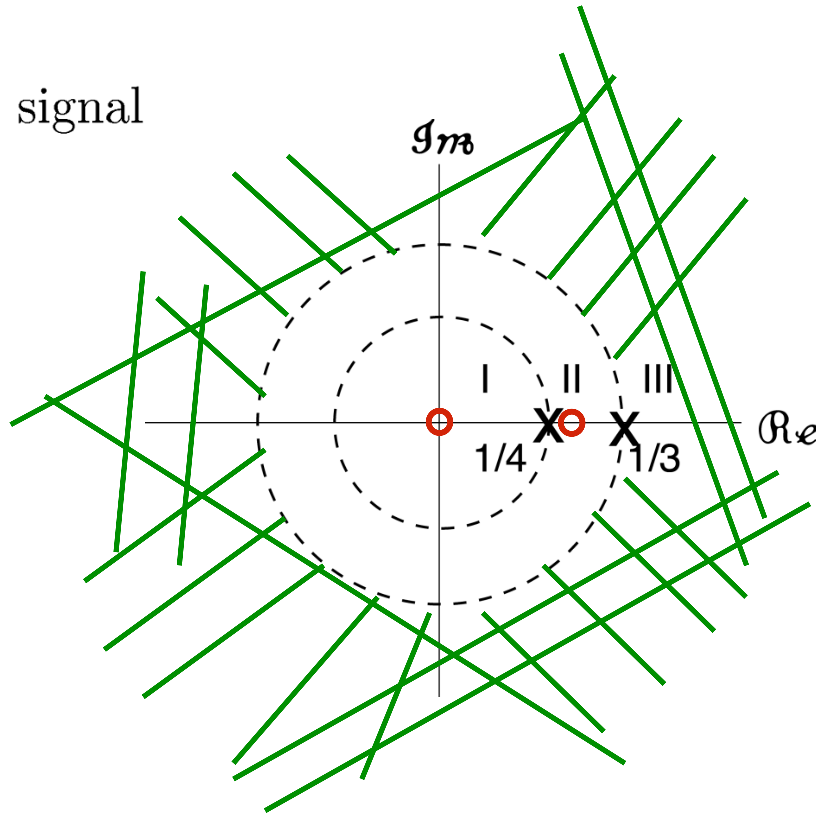
3 POSSIBLE SIGNALS IN TIME !!! 3 POSSIBLE SOLUTIONS

ROC III:

$$|z| > \frac{1}{3} \quad \text{- right-sided signal}$$

$$x_1[n] = \left(\frac{1}{4}\right)^n u[n]$$

$$x_2[n] = 2 \cdot \left(\frac{1}{3}\right)^n u[n]$$



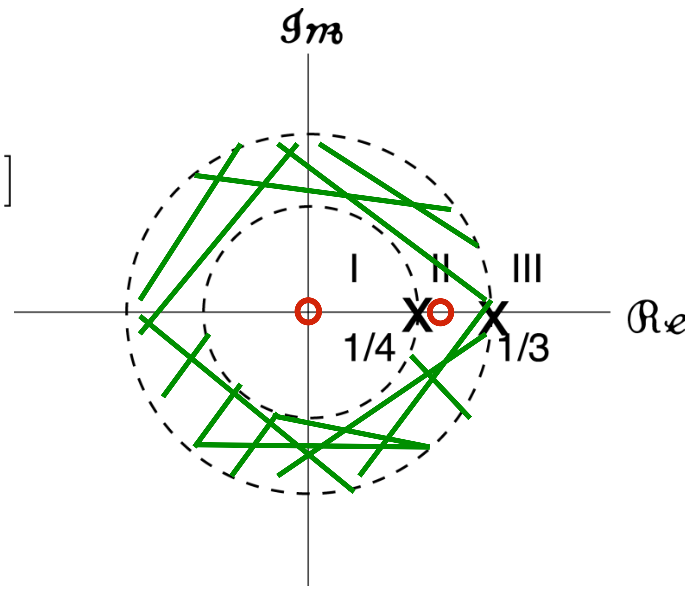
Example 14 - problem

3 POSSIBLE SIGNALS IN TIME !!! 3 POSSIBLE SOLUTIONS

ROC II: $\frac{1}{4} < |z| < \frac{1}{3}$ - two-sided signal

$$x_1[n] = \left(\frac{1}{4}\right)^n u[n]$$

$$x_2[n] = -2 \cdot \left(\frac{1}{3}\right)^n u[-n-1]$$



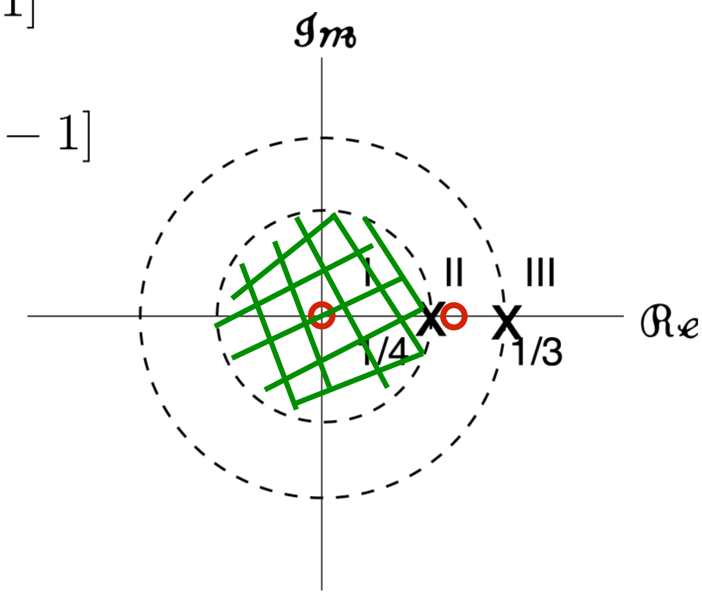
Example 14 - problem

3 POSSIBLE SIGNALS IN TIME !!! 3 POSSIBLE SOLUTIONS

ROC I: $|z| < \frac{1}{4}$ - left-sided signal

$$x_1[n] = -\left(\frac{1}{4}\right)^n u[-n-1]$$

$$x_2[n] = -2 \cdot \left(\frac{1}{3}\right)^n u[-n-1]$$



GENERAL IDEA

THE NUMBER OF POSSIBLE SIGNALS IN TIME IS:
NUMBER OF REAL POLES WITH DIFFERENT MODULE (and different from 0 and infinity) + 1

"…two poles complex conjugates generate only 2 possible signals…."

Example 15

$$X(z) = z^2 - z$$

↓

In this case is much easy: we know that corresponds to a finite-length signal (only one pole at infinity); the ROC is all the complex plane except infinity.

↓

$$x[n] = \delta[n+2] - \delta[n+1]$$

Example 16

$$X(z) = \frac{z^2 + 3z - 1}{z^2} \rightarrow X(z) = 1 + 3z^{-1} - z^{-2}$$

In this case is much easy: we know that corresponds to a finite-length signal (only one pole at zero); the ROC is all the complex plane except zero.

$$x[n] = \delta[n] + 3\delta[n-1] - \delta[n-2]$$

Example 17

$$X(z) = \frac{z^3 - 1}{z^2}$$

$$X(z) = z - z^{-2}$$

In this case is much easy: we know that corresponds to a finite-length signal (two poles at zero and infinity); the ROC is all the complex plane except zero and infinity.

$$x[n] = \delta[n+1] - \delta[n-2]$$

Example 18

$$X(z) = 3z^3 - z + 2z^{-4}$$

the ROC is all the complex plane except zero and infinity.

$$x[n] = 3\delta[n+3] - \delta[n+1] + 2\delta[n-4]$$

$$x[-3] = 3$$
$$x[-1] = -1$$
$$x[4] = 2$$
$x[n] = 0$, for the rest of values of *n*

UNILATERAL ZETA TRASFORM

UNILATERAL ZETA TRASFORM

$$X(z) = \sum_{n=0}^{+\infty} x[n]z^{-n}$$

It loses some properties and acquires others....

WE ONLY CONSIDER THE BILATERAL ZETA TRANSFORM:

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n]z^{-n}$$