Topic 1- part 6 - Why a “transformed” domain? for signal in discrete time (DT)

Discrete Time Systems (DTS)

Luca Martino — luca.martino@urjc.es — http://www.lucamartino.altervista.org

Why a transformed domain?

- Why do we “pass” to another domain different from the time domain?
- the answer in the next slides (at least the first main reason)… for signal in discrete time (DT)

Why a transformed domain?

- **HERE FIRST MAIN EXPLANATION/MOTIVATION: system point of view, eigenfunctions and eigenvalues…**
- **The second main reason (in other slides): signal point of view, spectral analysis, signal decomposition…**

Recall eigenvalues/eigenvectors

$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$$

- eigenvector
- eigenvalue

Why a transformed domain?

- In discrete time, very important signals are

$$z^n$$

- we will see that many signals in DT can be expressed as linear combination of z^n

Why a transformed domain?

- The variable "z" is a complex number,

$$z^n = (re^{j\Omega})^n$$

- This "physical" interpretation is the reason for the second main reason that we will see in other slides…

Why a transformed domain?

- Let us consider an LTI system:

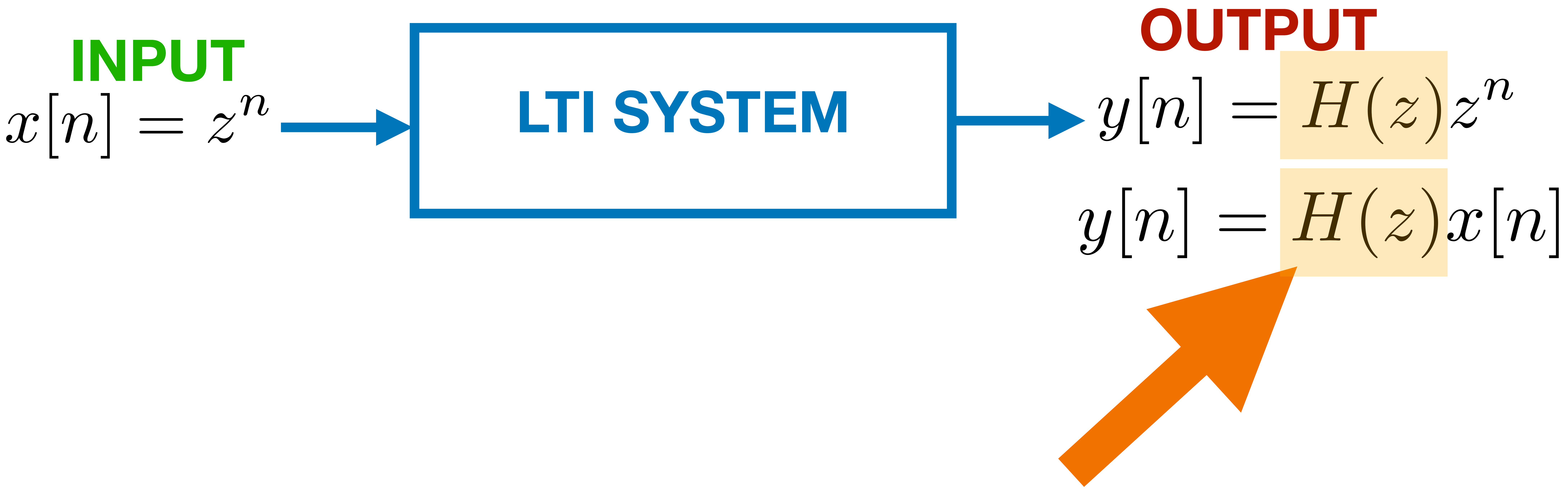
$$x[n] = z^n$$

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=-\infty}^{\infty} h[k]z^{n-k} = z^n \sum_{k=-\infty}^{\infty} h[k]z^{-k}$$

$$y[n] = z^n H(z)$$

ZETA TRANSFORM OF THE IMPULSE RESPONSE

Why a transformed domain?



ZETA TRANSFORM OF THE IMPULSE RESPONSE

Why a transformed domain?

$$H(z) = \text{eigenvalue}$$

$$z^n = \text{eigenfunction}$$

ZETA Transform

- **ZETA Transform of the impulse response:**

$$H(z) = \sum_{k=-\infty}^{\infty} h[k]z^{-k}$$

- **Function again defined in the complex plane (z is a complex number).**

Zeta Transform

- Zeta transformation of the a generic function/signal:

$$X(z) = \sum_{k=-\infty}^{\infty} x[k] z^{-k}$$

Zeta Transform

- Zeta transformation is defined in the complex domain and takes complex values:

$$z \in \mathbb{C} \qquad X(z) \in \mathbb{C}$$

then usually the people plot and study:

$$|X(z)| \qquad \text{Real}[X(z)]$$

$$\text{phase}\ [X(z)] \qquad \text{Imag}[X(z)]$$

Why a transformed domain?

- Hence, given:

$$x[n] = z^n$$

- then the output is:

$$y[n] = H(z)x[n]$$
$$|y[n]| = |H(z)||x[n]|$$

Why a transformed domain?

$$y[n] = H(z)x[n]$$

$$|y[n]| = |H(z)||x[n]|$$

- Then, when:

$$|H(z)| \to 0 \qquad |y[n]| = 0$$

$$|H(z)| \to \infty \quad |y[n]| \to \infty$$

Why a transformed domain?

- The ZETA transform of the impulse response h(t) says almost everything regarding the LTI system:

We want to study then

$$H(z)$$

as function of z.

Just to see if you understand....

$$\sum_{k=-\infty}^{\infty} h[k]{\color{red}(1+j)}^{-k} = ?$$

$$H(1+j)$$

Questions?