Topic 1- part 7 - Why a “transformed” domain? second main reason…

Discrete Time Systems (DTS)

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Why a transformed domain?

- Why do we “pass” to another domain different from the time domain?
- We have seen the first main reason…

Why a transformed domain?

- **The second main reason (in other slides): signal point of view, spectral analysis, signal decomposition…**

(SIGNAL) DECOMPOSITION

Decomposition = linear combinations of bases

Numbers in "10-basis":

$$\text{Number} = \sum_{n=0}^{K} a_n (10)^n$$

$$126 = (1 \times 100) + (2 \times 10) + (6 \times 1)$$

$$126 = (1 \times 10^2) + (2 \times 10^1) + (6 \times 10^0)$$

Basis...

Coordinates/Components
(according to "this space" - this basis...)

Decomposition = linear combinations of bases

Numbers in "2-basis":

In binary, numbers are represented similarly, except that the base is now 2 rather than 10. So the number 13 would be written as

Coordinates/Components (according to "this space" - this basis...)

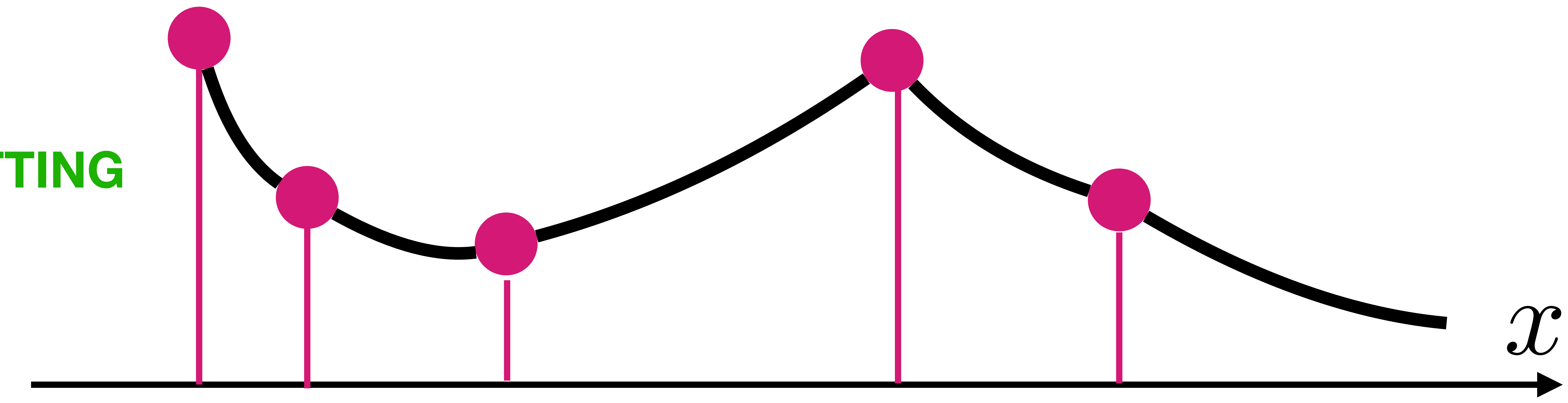
$$13 = (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$$

Basis...

Which means that the binary representation of 13 in binary is simply 1101, where each column represents the coefficient for the powers of 2, similar to the way things work in base 10.

Decomposition = linear combinations of bases

POLYNOMIAL DATA FITTING



$x^0 = 1$

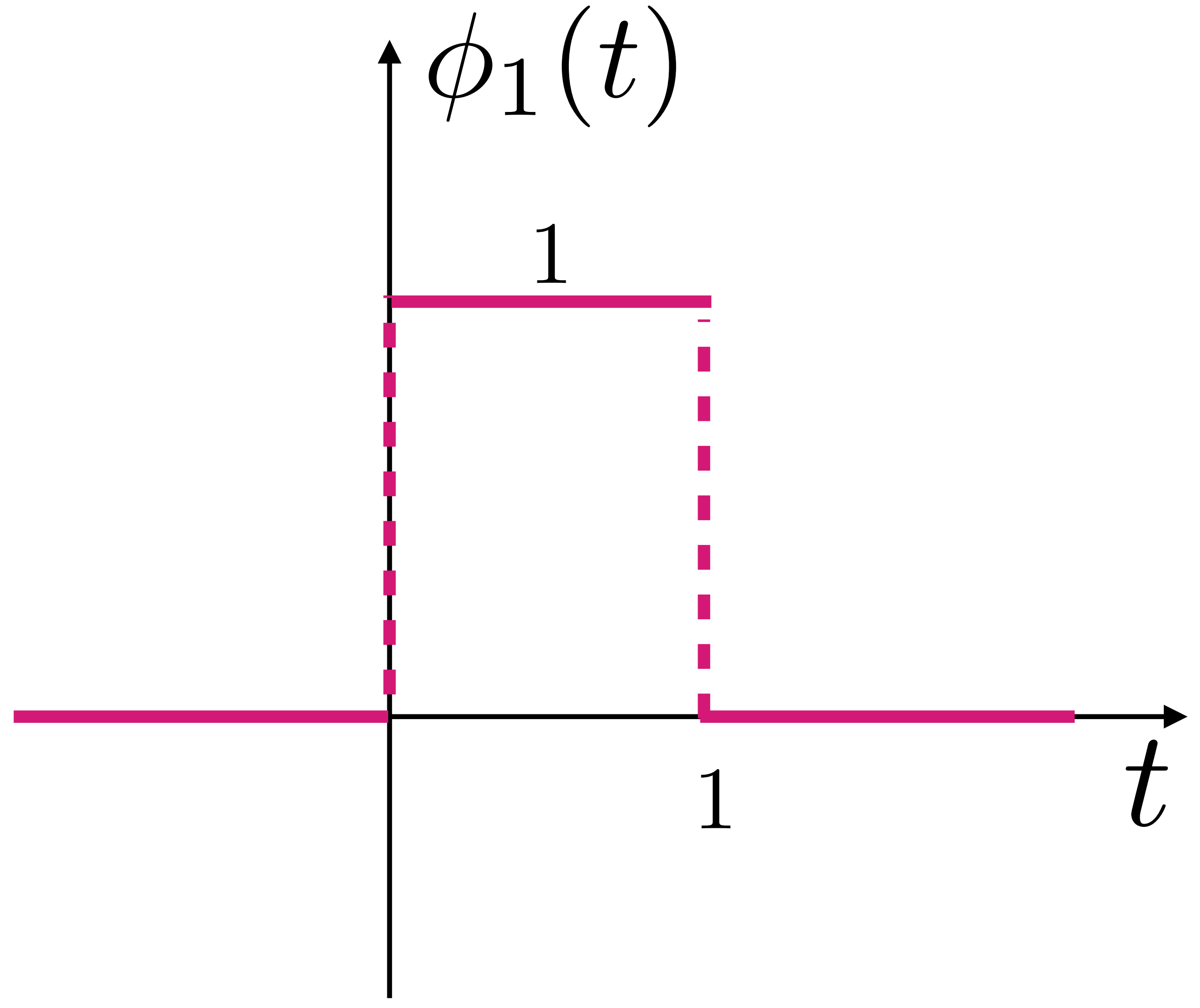
$$y = \sum_{n=0}^{K} a_n x^n = a_0 + a_1 x + a_2 x^2 + \ldots + a_K x^K$$

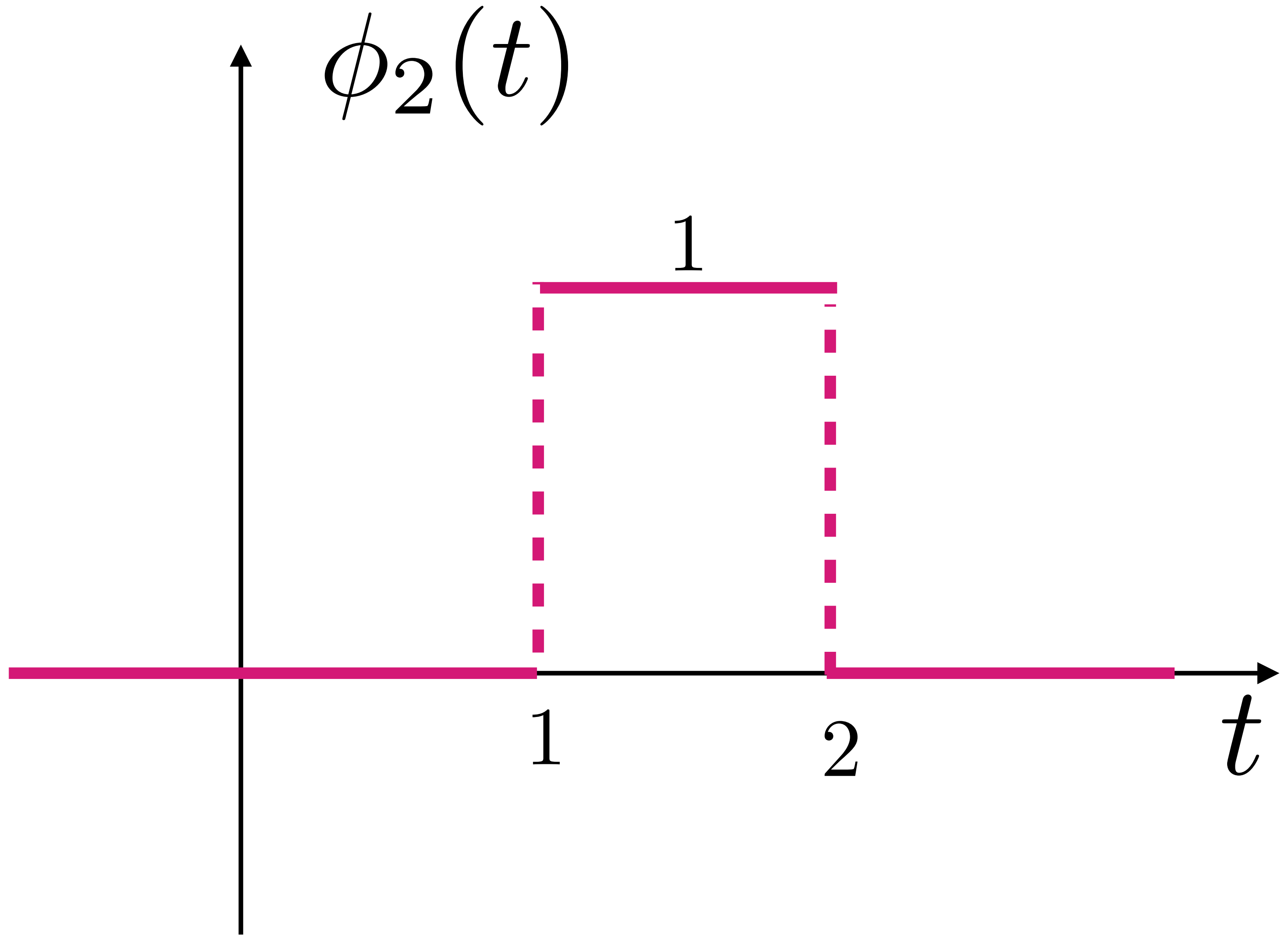
Basis...

Coordinates/Components
(according to “this space” - this basis…)

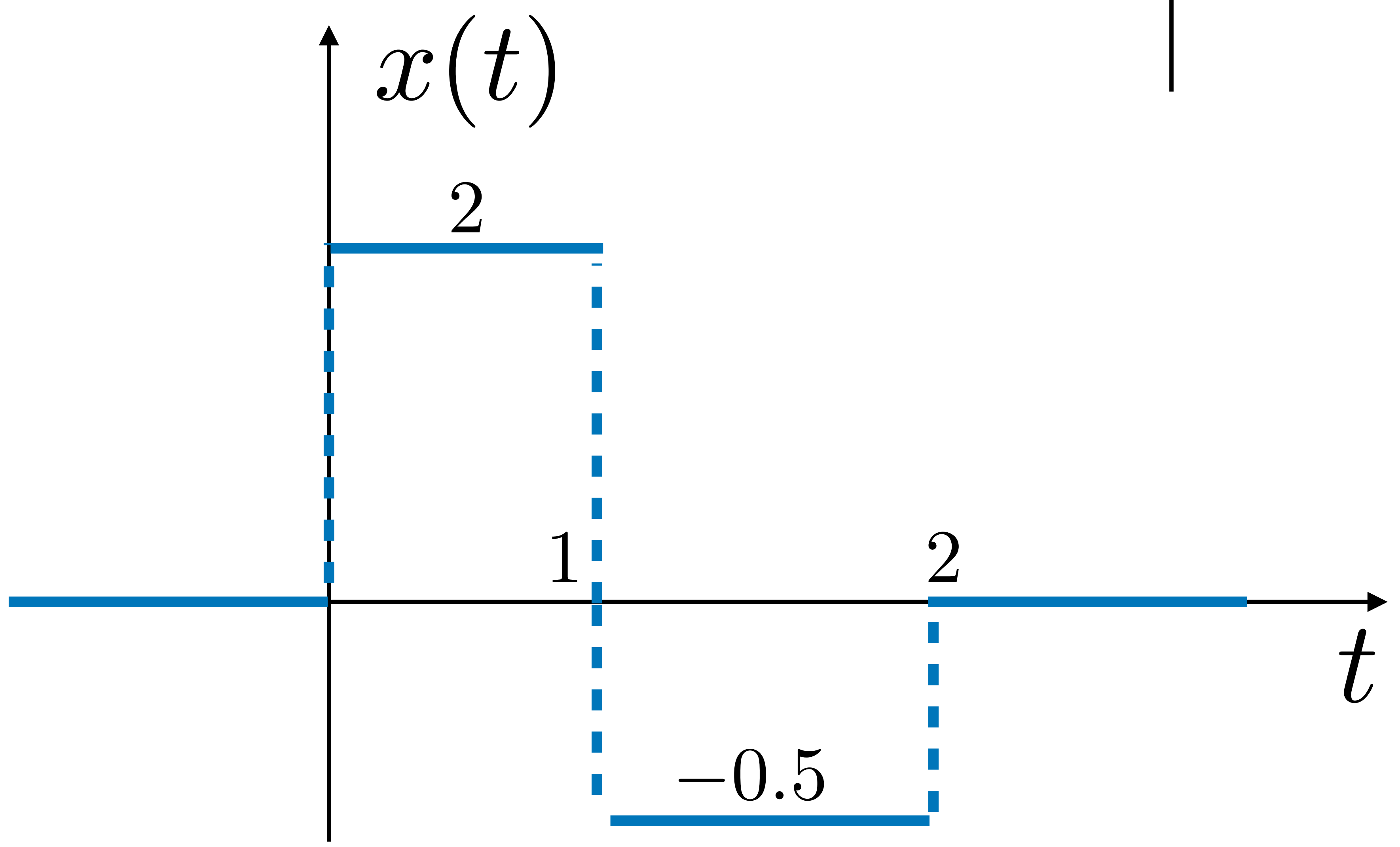
Decomposition = linear combinations of bases

Considering the basis:

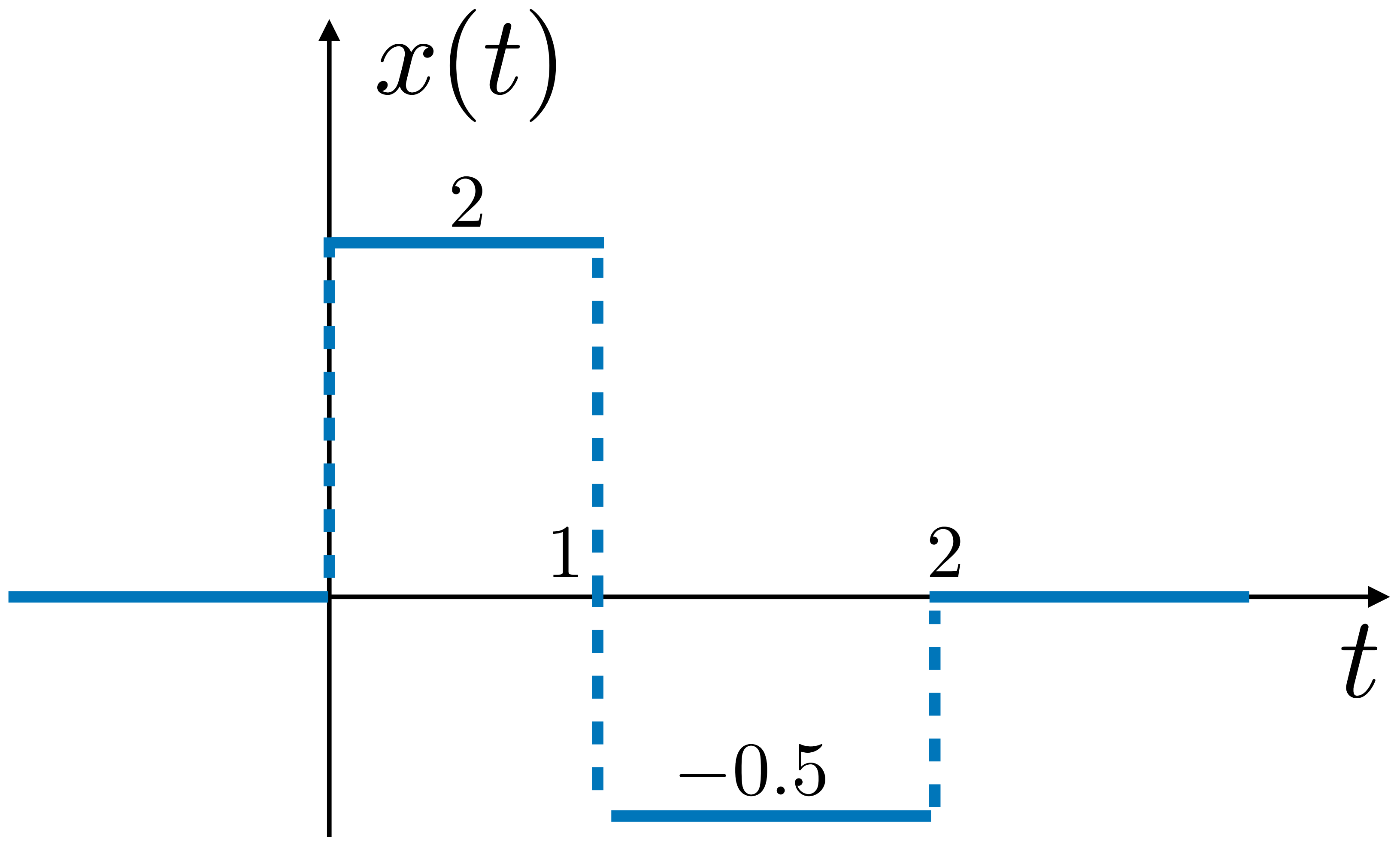




and the signal:



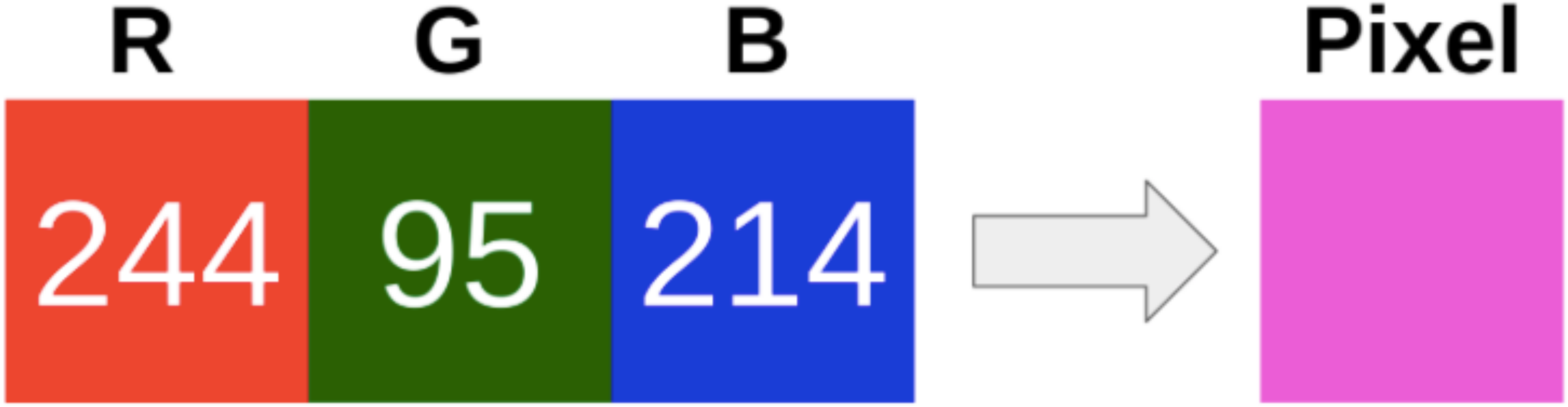
Decomposition = linear combinations of bases



$$= 2\phi_1(t) - 0.5\phi_2(t)$$

Coordinates/Components
(according to “this space” - this basis…)

Decomposition = linear combinations of bases

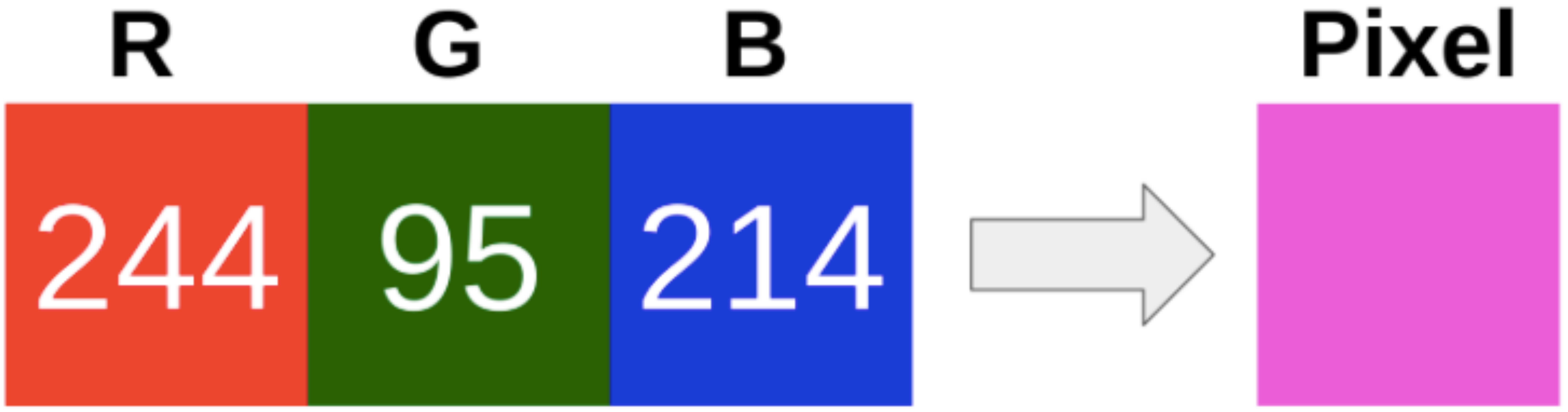


$$\text{Color-pixel} = 244\times\ \text{Red} + 95\times\ \text{Green} + 214\times\ \text{Blue}$$

Basis…

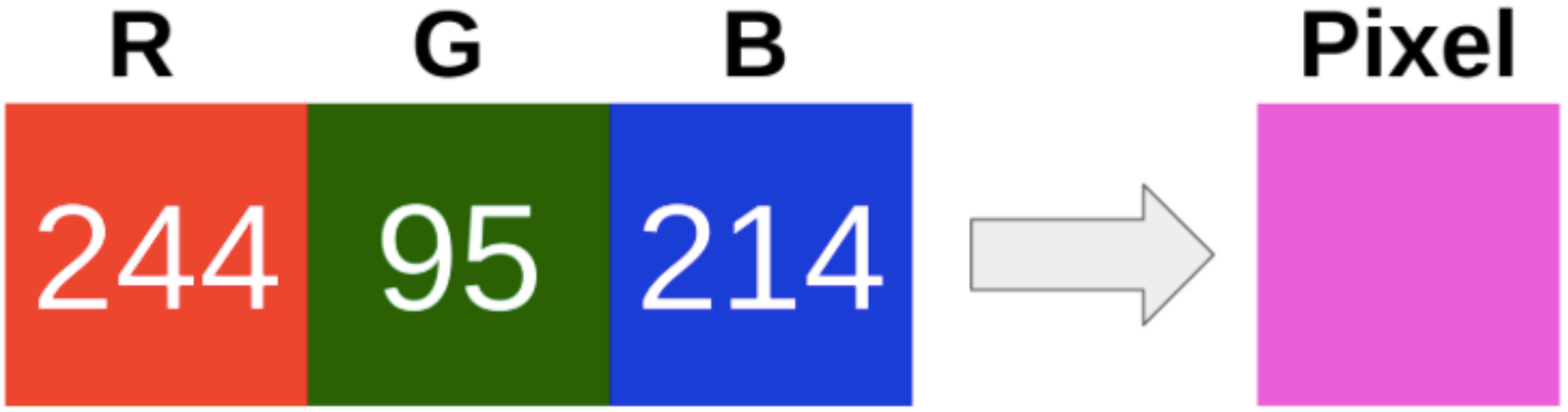
Coordinates/Components
(according to “this space” - this basis…)

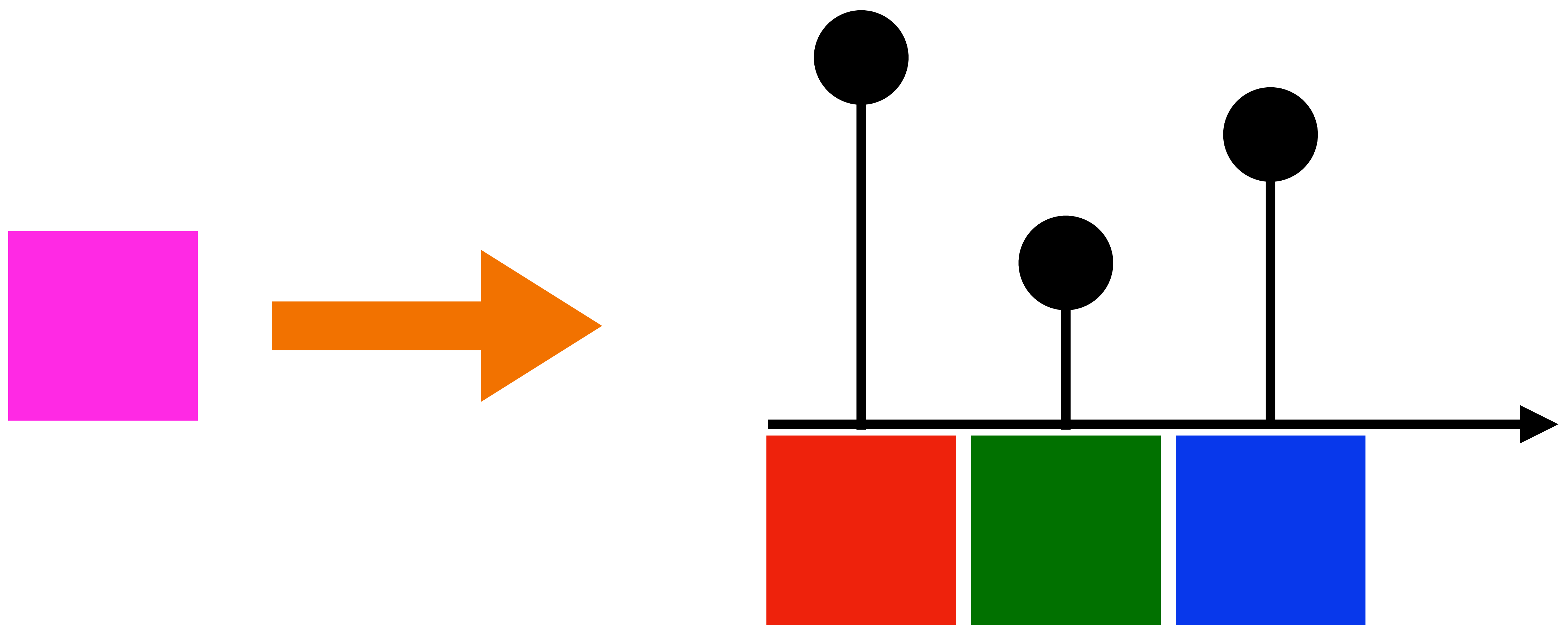
Decomposition = linear combinations of bases



Hence, we can say
"How much *red* we have inside *magenta* ?"
We have a lot of red, more than blue, and no too much green…

Decomposition = linear combinations of bases



Decomposition of 

Decomposition = linear combinations of bases

Ingredients in a recipe !

Signal

=

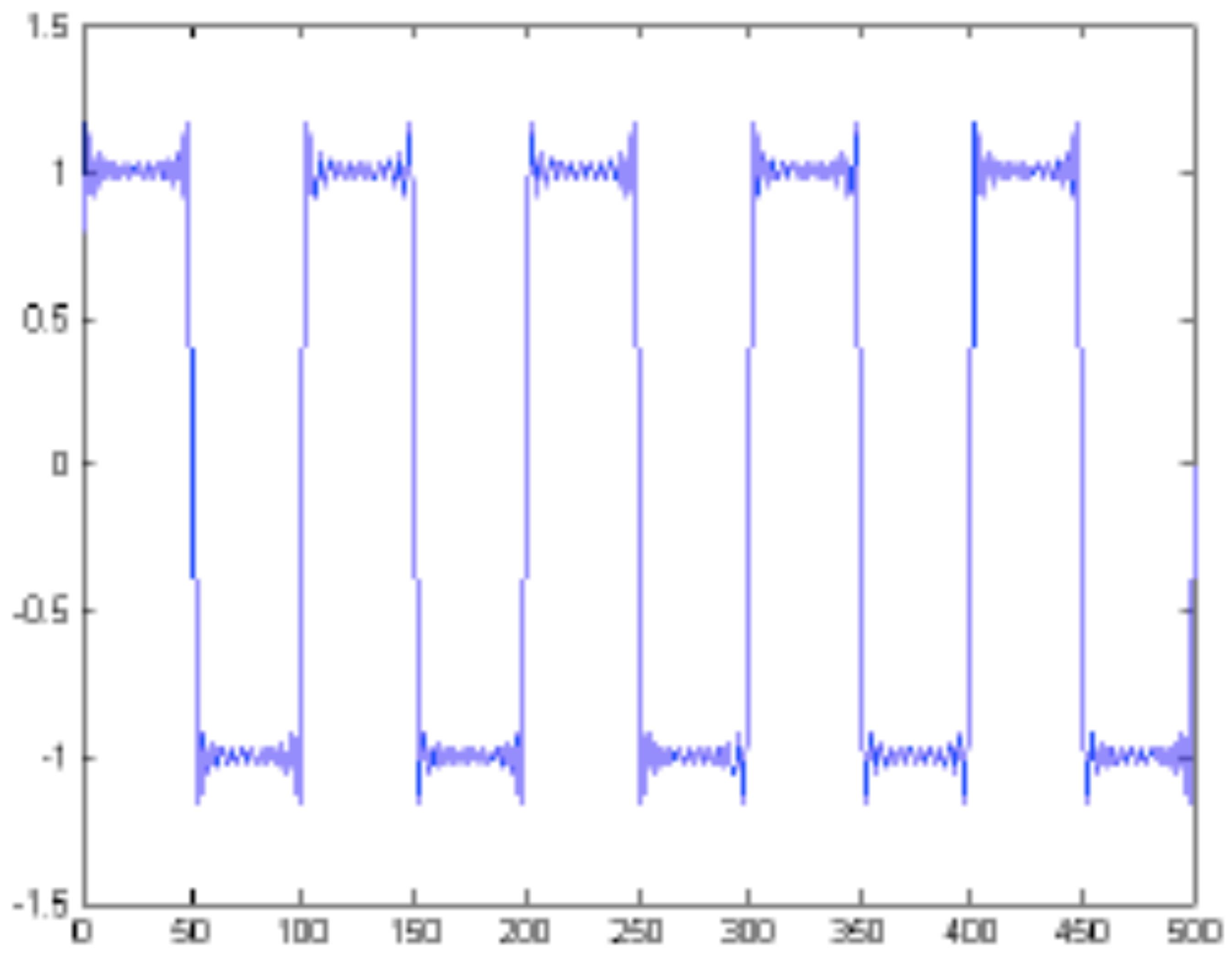
Components !!

Decomposition = linear combinations of bases

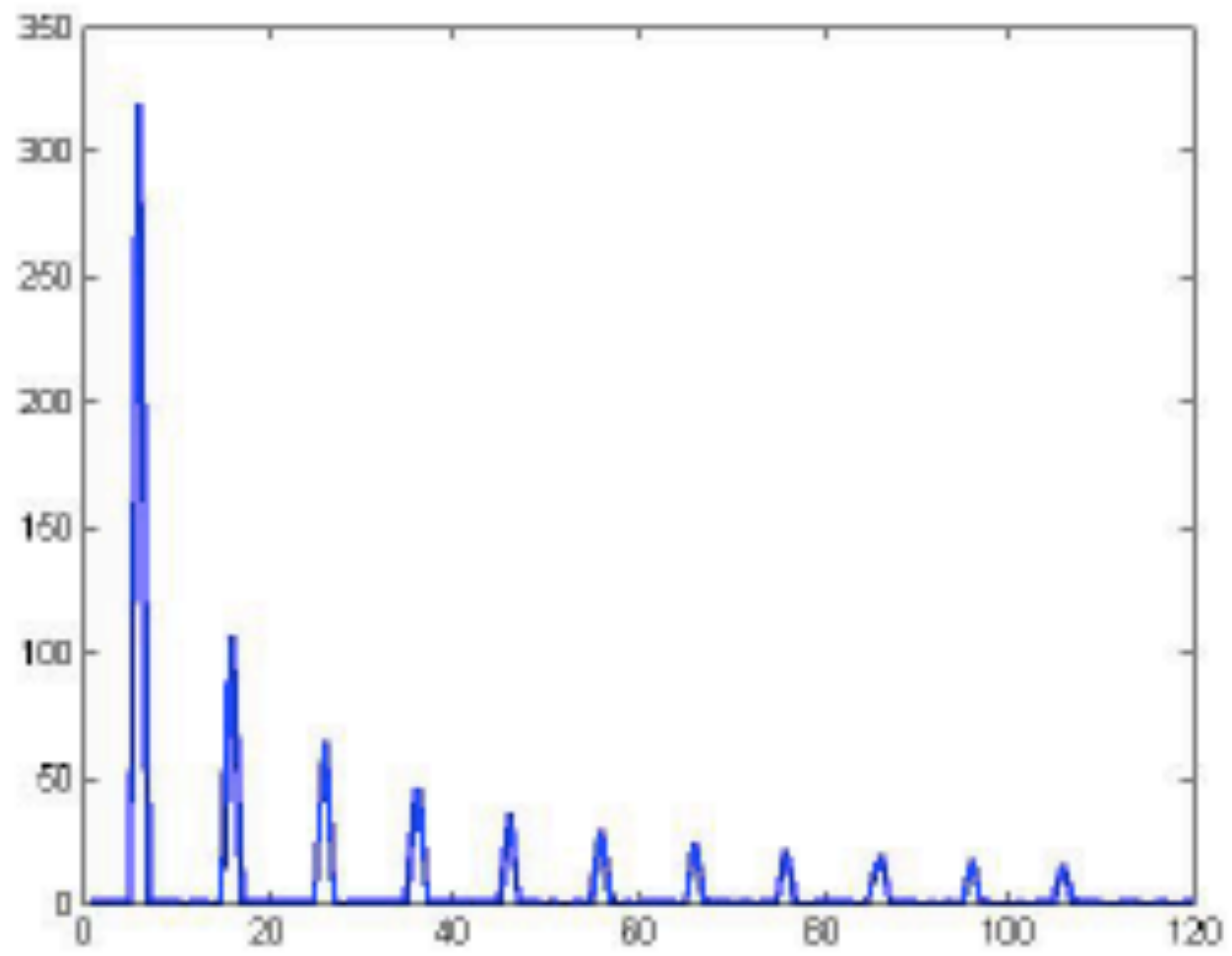
We want to find the "ingredients" that form a given signal - i.e., find the "ingredients" to recover/redo/reconstruct the given signal….

“Dividing-expressing” a signal in components of different frequencies

- ¿Qué hay detrás de una señal? …
  - Diversas componentes de frecuencia y amplitud



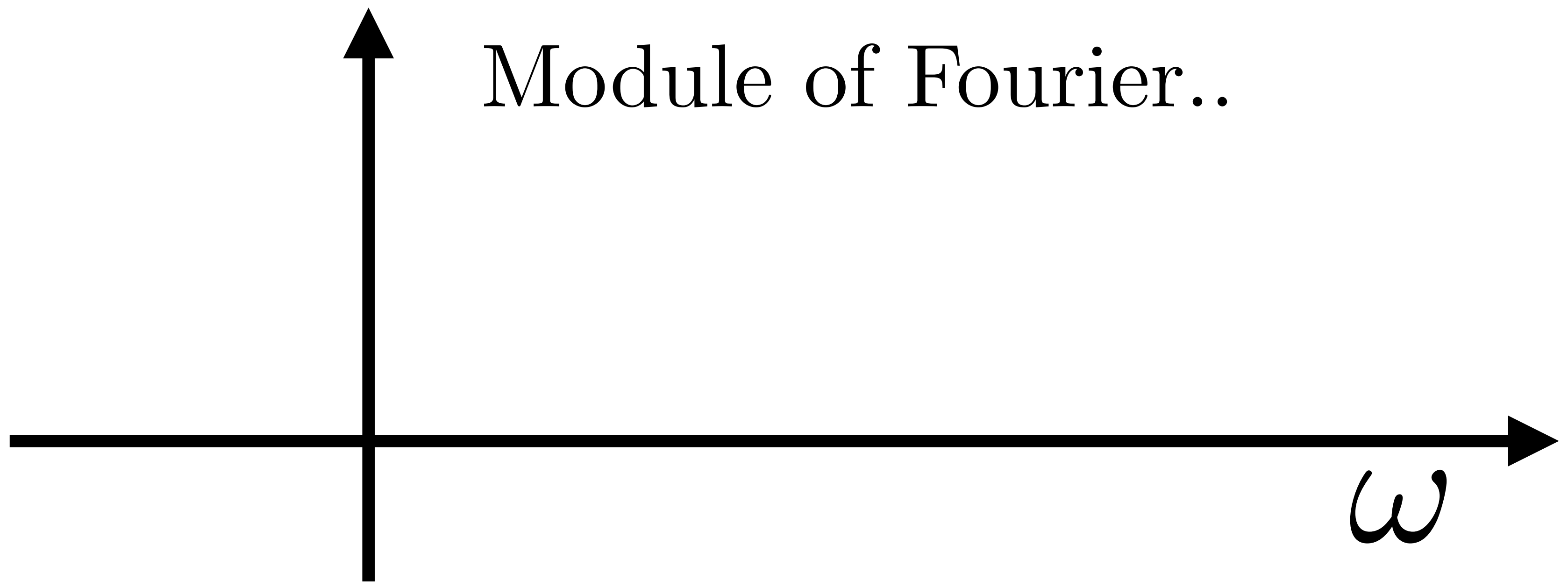
Dominio del tiempo
(continuo o discreto)



Dominio de la frecuencia

Basic idea: express a signal as a (finite or infinite) sum of sinusoidal functions

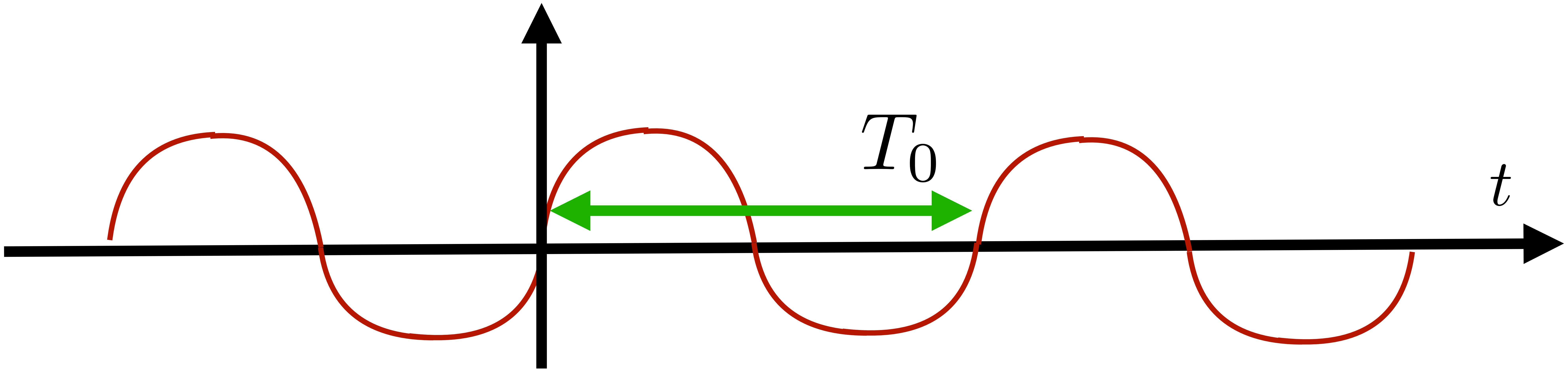
Spectral analysis



Basic idea: express a signal as a (finite or infinite) sum of sinusoidal functions with different frequencies

(Concept of) FREQUENCY

Concept of Frequency



$$f_0 = \frac{1}{T_0} \quad \text{(Hz)}$$

$$\omega_0 = \frac{2\pi}{T_0} \quad \text{(rad/sec)}$$

$$\omega_0 = 2\pi f_0$$

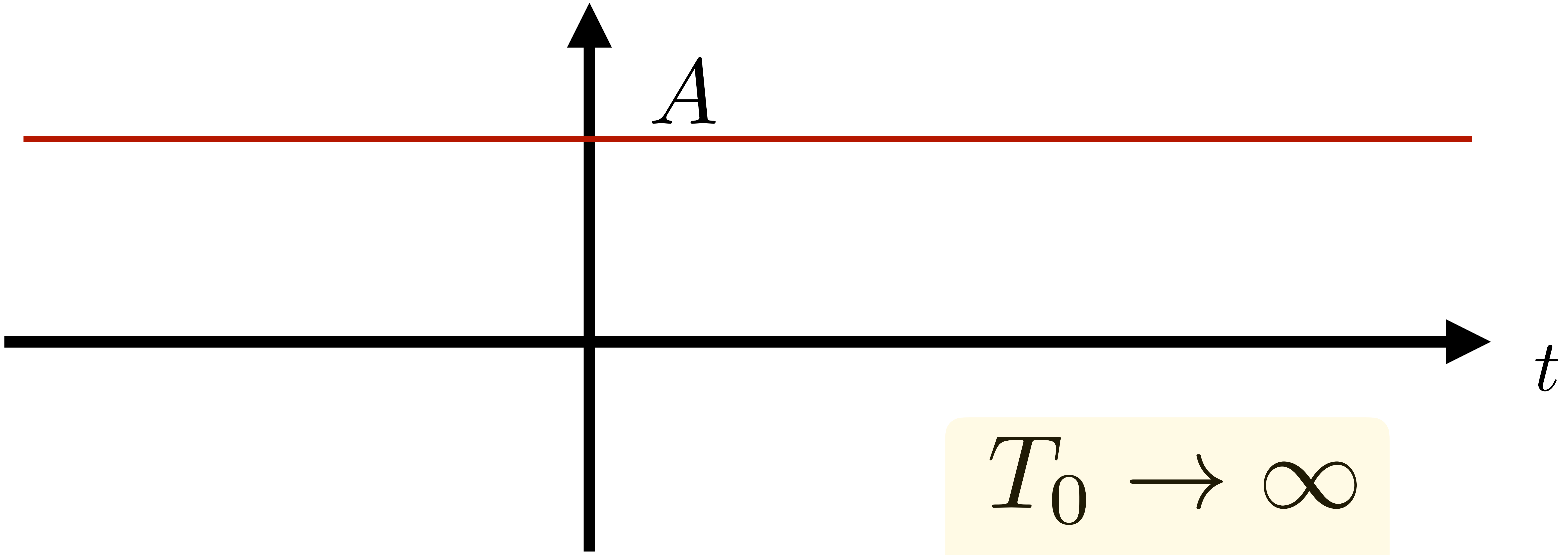
Concept of Frequency

$$\omega = \frac{2\pi}{T} = 2\pi f,$$

where:

- ω is the angular frequency (measured in radians per second),
- T is the period (measured in seconds),
- f is the ordinary frequency (measured in hertz) (sometimes symbolised with ν).

Concept of Frequency: NULL FREQUENCY



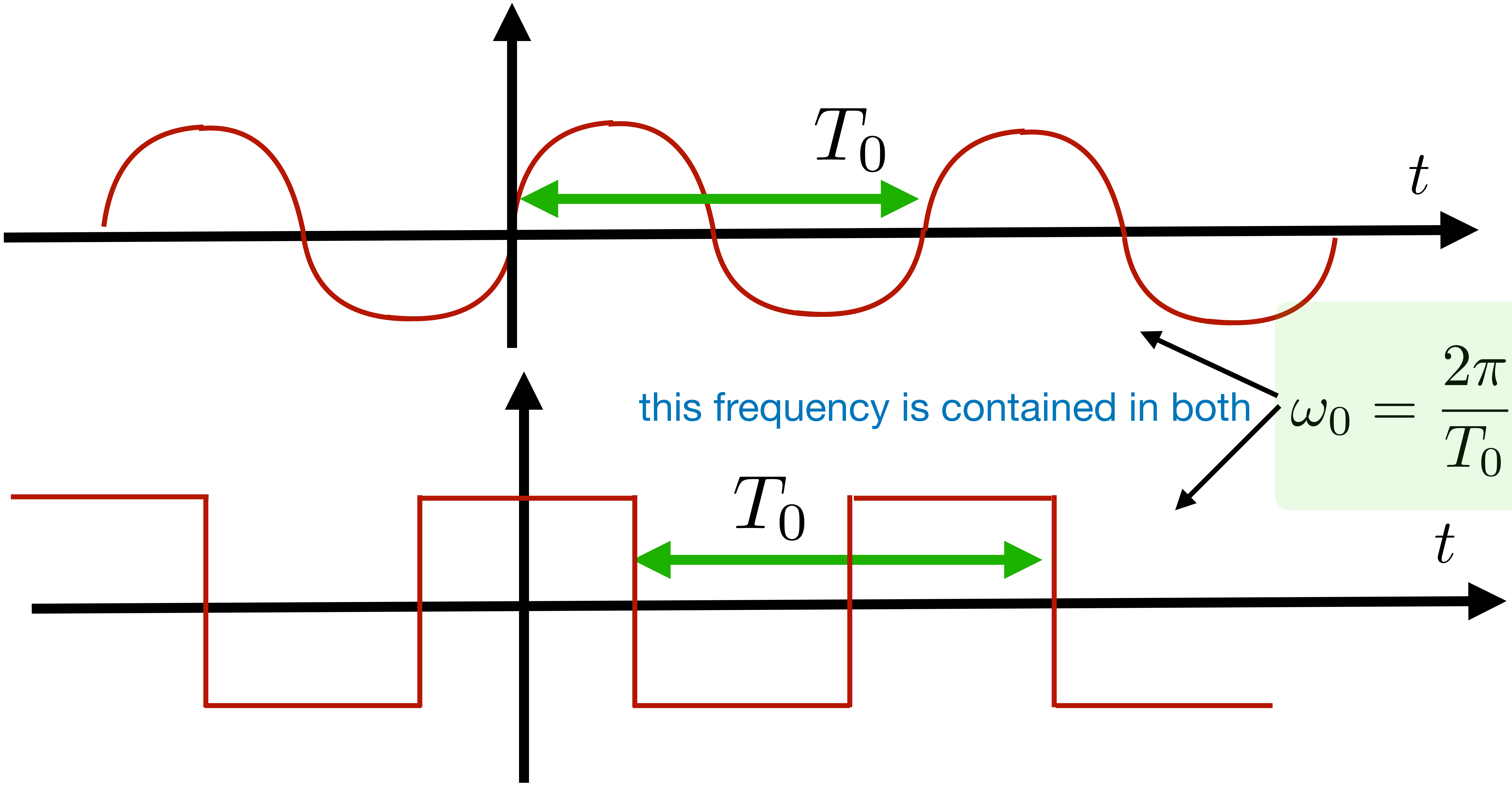
$$T_0 \to \infty$$
$$\omega_0 = 0$$

Constant Signal ==> contains only the null frequency

Concept of Frequency: NULL FREQUENCY

Hence, each signal with non-zero mean contains (at least) the null frequency

Concept of Frequency

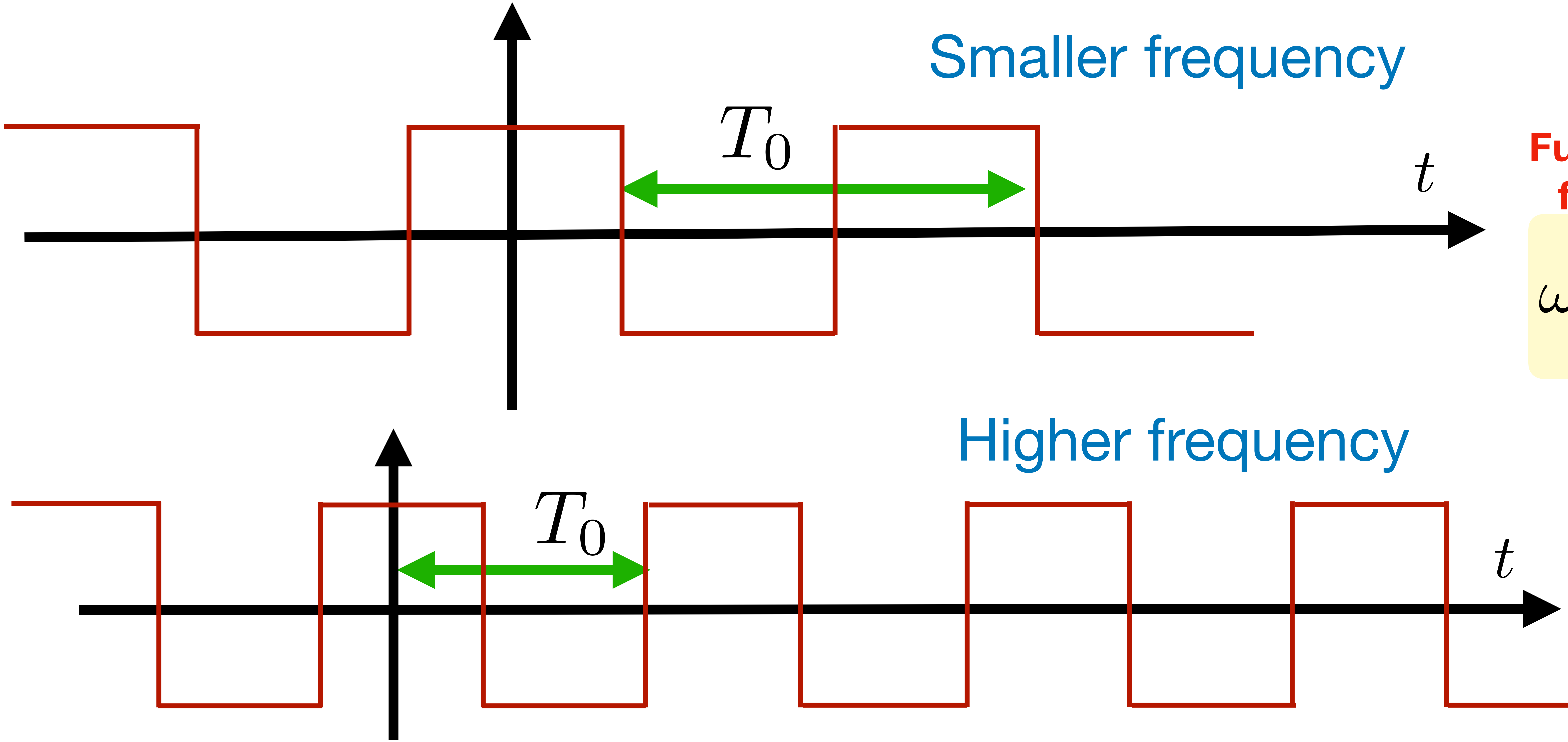


$$\omega_0 = \frac{2\pi}{T_0} \quad (\mathrm{rad/sec})$$

For the **(MAIN) FUNDAMENTAL FREQUENCY w0,** the shape does not matter…

Concept of Frequency

Frequency ==> Oscillations



Fundamental frequency

$$\omega_0 = \frac{2\pi}{T_0}$$

PERIODIC signals in CT or DT

The periodic signals (with period T or N) contain their fundamental frequency and the multiple of this fundamental frequency.

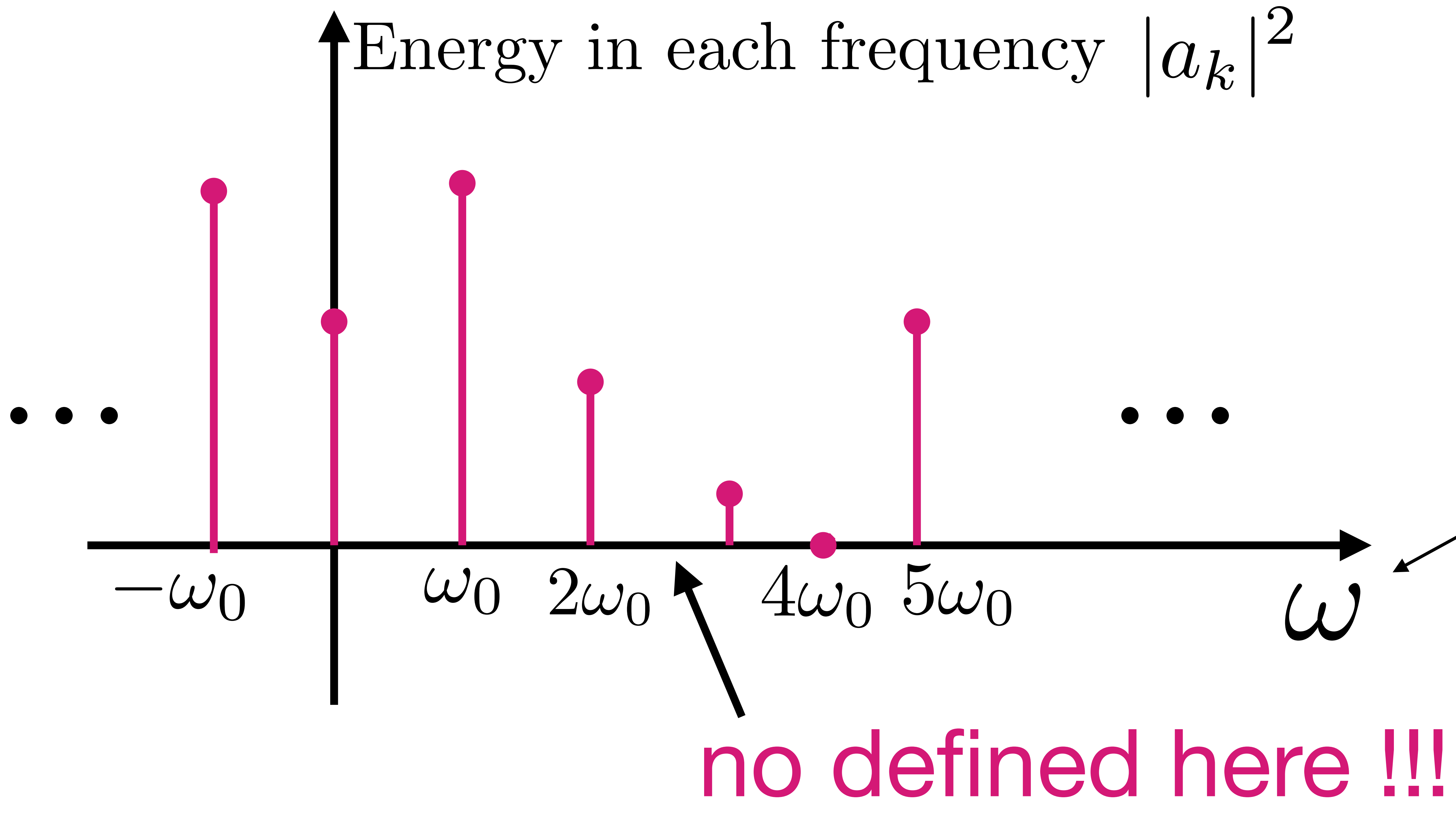
$$\omega_0 = \frac{2\pi}{T} \quad \longrightarrow \quad k\omega_0$$

$$\Omega_0 = \frac{2\pi}{N} \quad \longrightarrow \quad k\Omega_0$$

$$k = ... - 2, -1, 0, 1, 2, ...$$

PERIODIC signals in CT or DT

The periodic signals contain their fundamental frequency and the multiple of this fundamental frequency.



Power *spectrum* of the signal

PERIODIC signals in CT or DT

Fourier Series
$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

$$\cos(\omega_0 t) = \frac{1}{2}[e^{j\omega_0 t} + e^{-j\omega_0 t}] \rightarrow$$

$$a_1 = \frac{1}{2}, \qquad a_{-1} = \frac{1}{2},$$

$$a_k = 0 \text{ for all } k \neq -1, 1$$

$$\sin(\omega_0 t) = \frac{1}{2j}[e^{j\omega_0 t} - e^{-j\omega_0 t}] \rightarrow$$

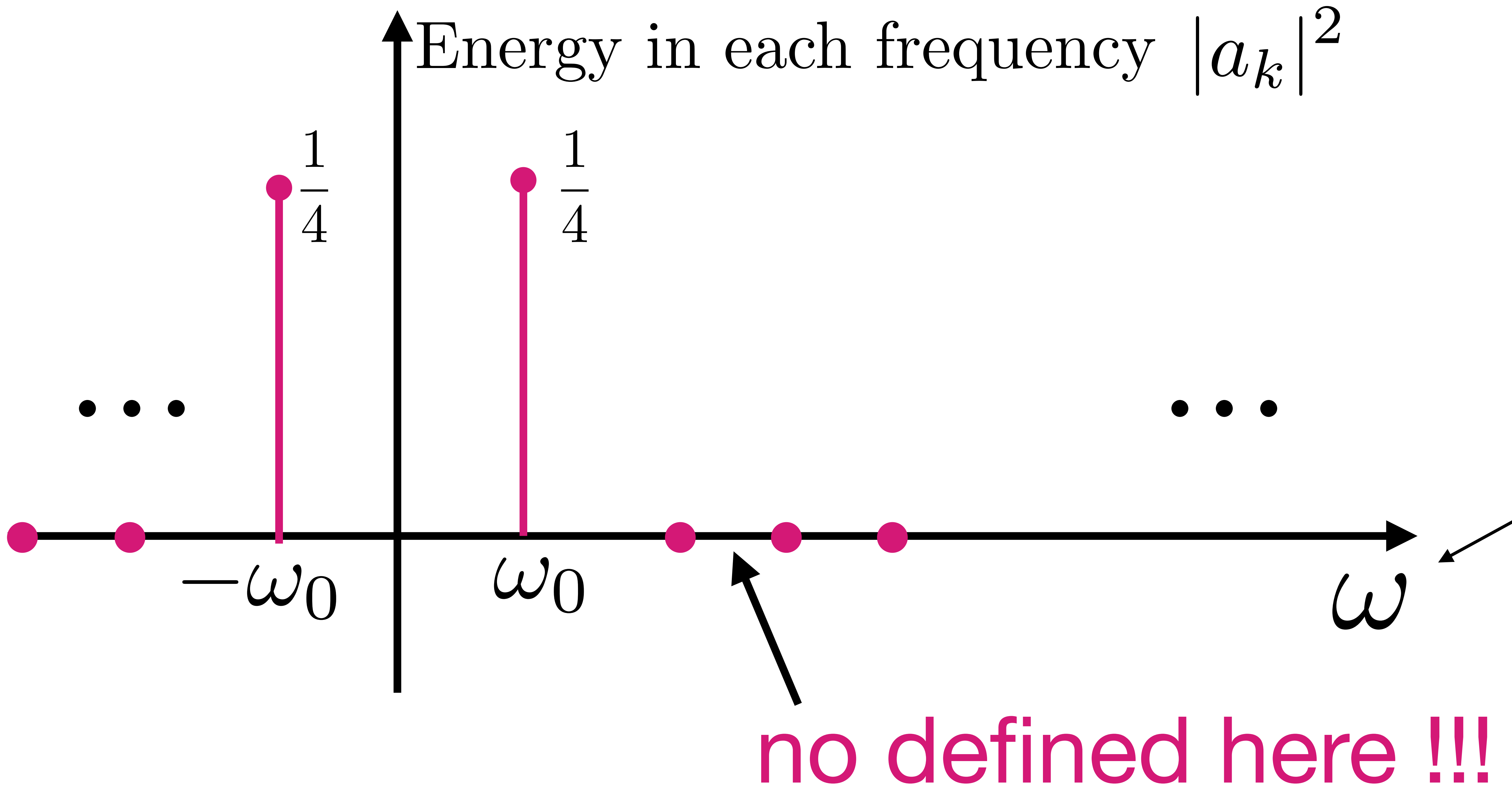
$$a_1 = \frac{1}{2j}, \qquad a_{-1} = -\frac{1}{2j},$$

$$a_k = 0 \text{ for all } k \neq -1, 1$$

PERIODIC signals in CT or DT

Spectrum of $\sin(\omega_0 t)$

$\cos(\omega_0 t)$



Power *spectrum* of the signal

Frecuencias negativas

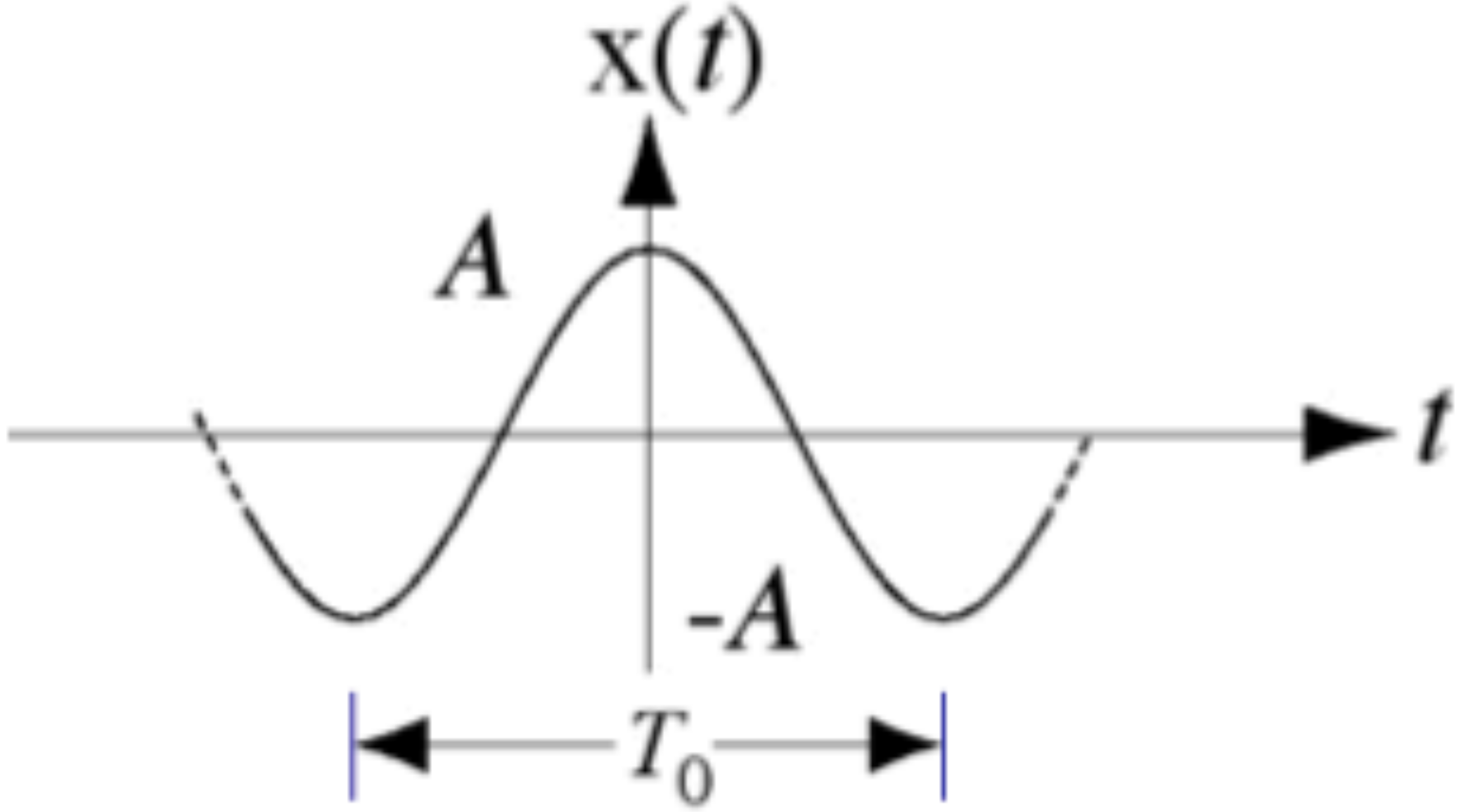
Una sinusoide puede describirse matemáticamente de varias formas:

$$x(t) = A\cos\left(\frac{2\pi t}{T_0}\right) = A\cos(\omega_0 t)$$

$$x(t) = A\cos(-\omega_0 t)$$

$$x(t) = A\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$x(t) = A_1\cos(\omega_0 t) + A_2\cos(-\omega_0 t) \quad , \quad A_1 + A_2 = A$$

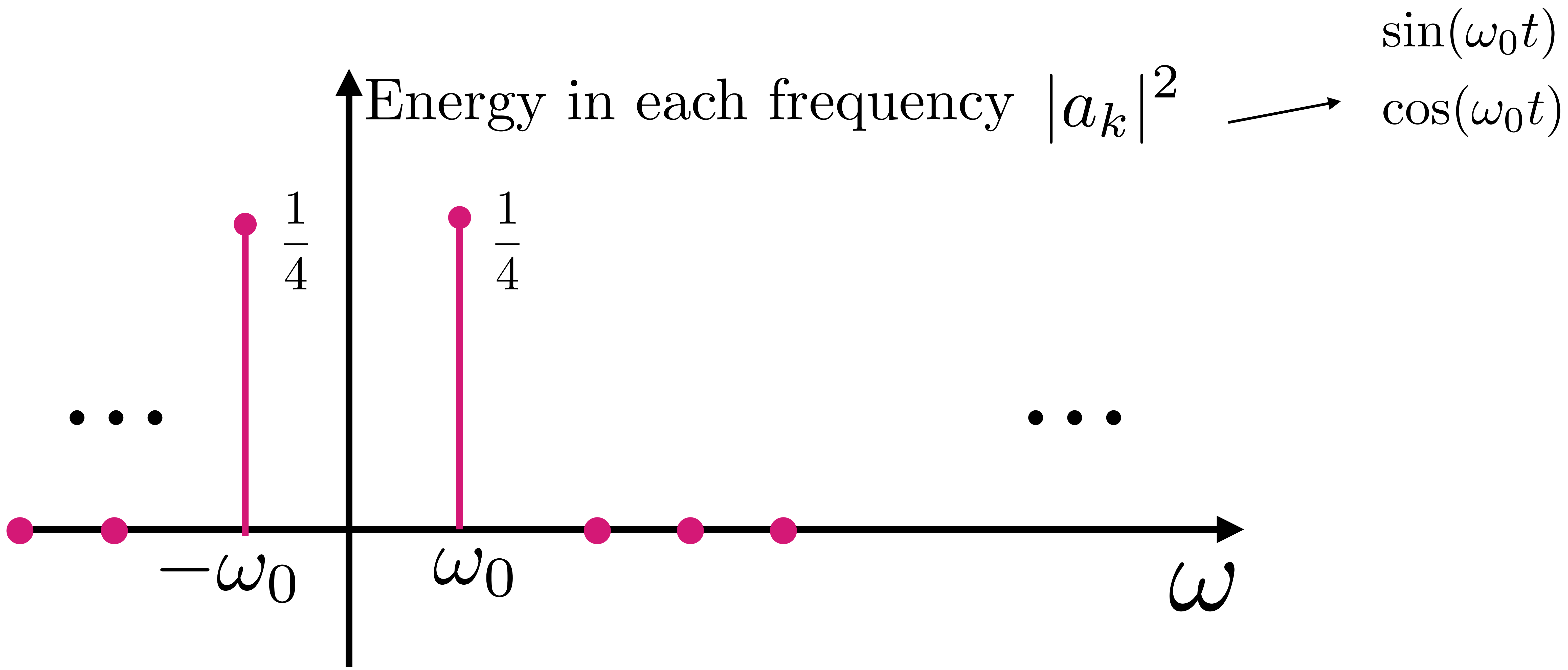


Y se puede representar de otras formas distintas …

Así pues, podríamos considerar que la frecuencia sea positiva o negativa. Desde el punto de vista del análisis de señal, no importa

Negative frequencies

No problem with negative frequencies: real (non-complex) signals (like data, measurements) have even-symmetric module of Fourier series/transform



PERIODIC signals in CT or DT

Fourier Series $$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

$$x(t) = e^{j\omega_0 t}$$

$\rightarrow$

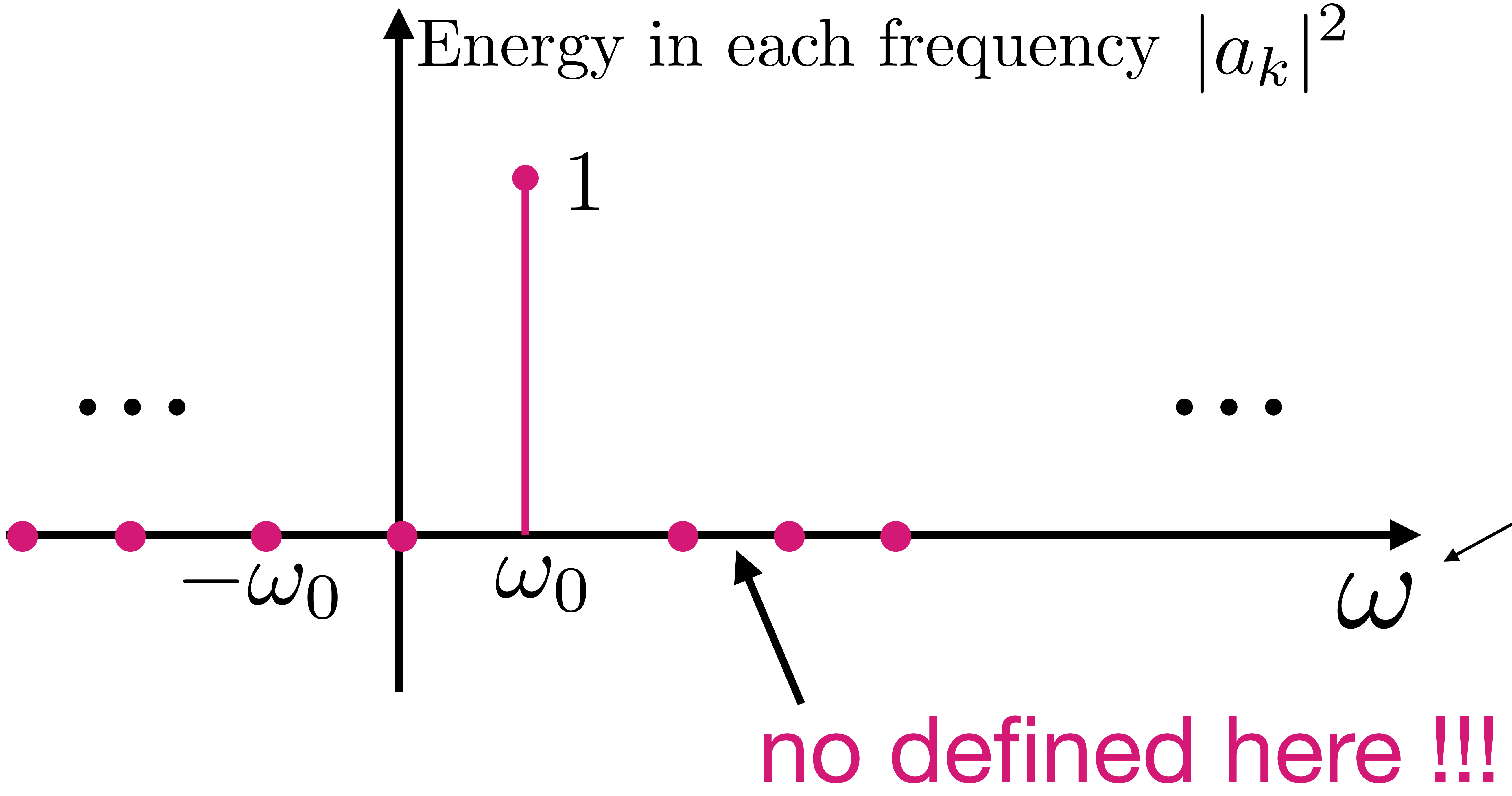
$$a_1 = 1$$

$$a_k = 0 \text{ for all } k \neq 1$$

PERIODIC signals in CT or DT

Spectrum of $\exp(j\omega_0 t)$

Since the signal is complex we have not even-symmetric of the Fourier module…



Power *spectrum* of the signal

Frequencies *contained in* a signal?

What is the meaning of frequency *contained in* a signal?

Consider the *non-periodic* signal but can be expressed as sum of two periodic signals:

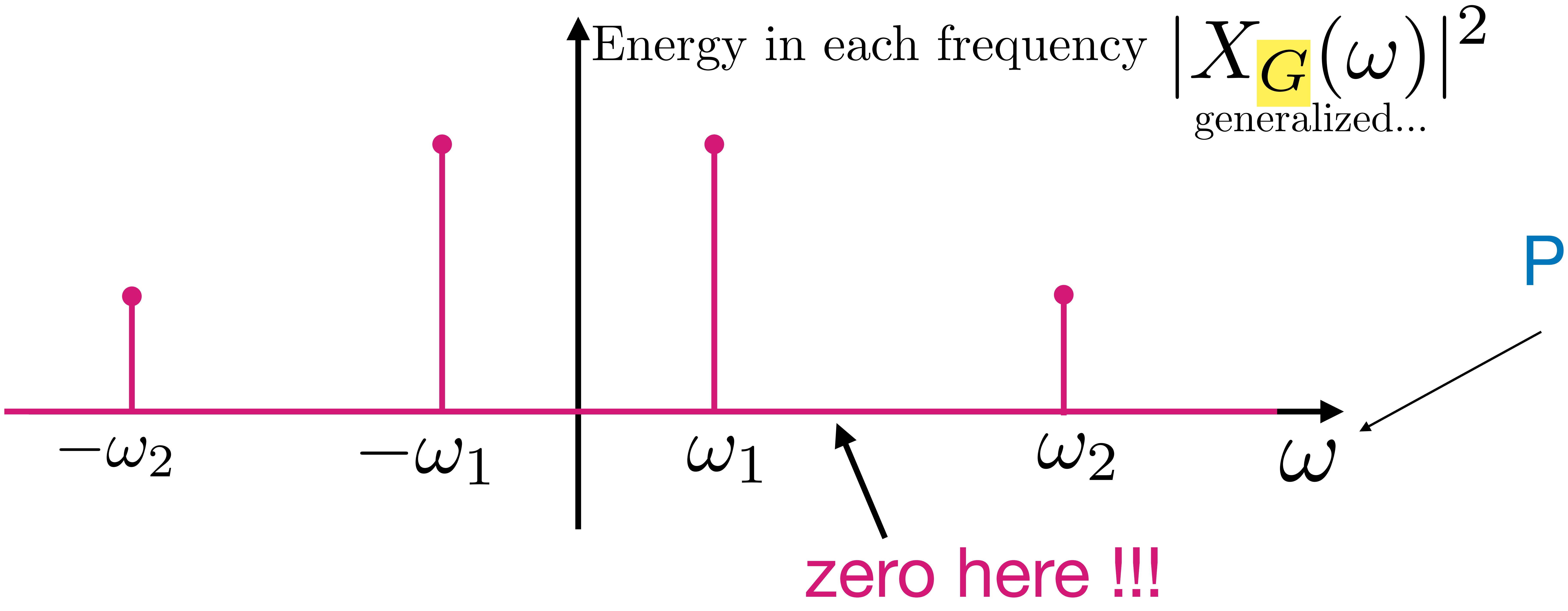
$$x(t) = \sin(\omega_1 t) + \cos(\omega_2 t)$$

The *non-periodic* signal x(t) contains the frequency w_1 and w_2 (and also -w_1, -w_2).

Frequencies *contained in* a signal?

What is the meaning of frequency *contained in* a signal?

$$x(t) = \sin(\omega_1 t) + \cos(\omega_2 t)$$



Power *spectrum* of the signal

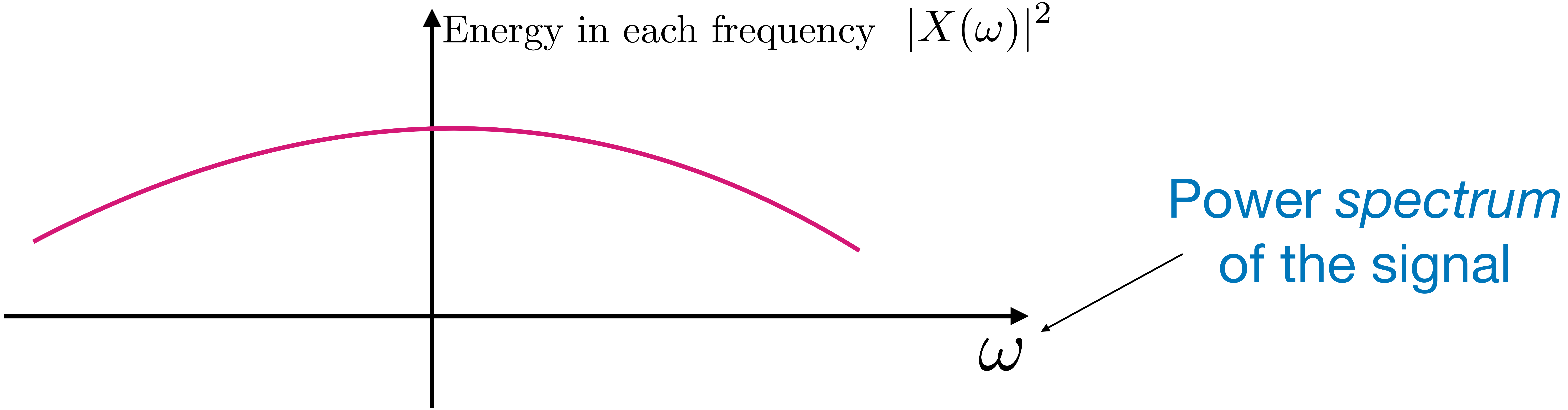
Frequencies *contained in* a signal?

$$x(t) = \sin(\omega_1 t) + \cos(\omega_2 t)$$

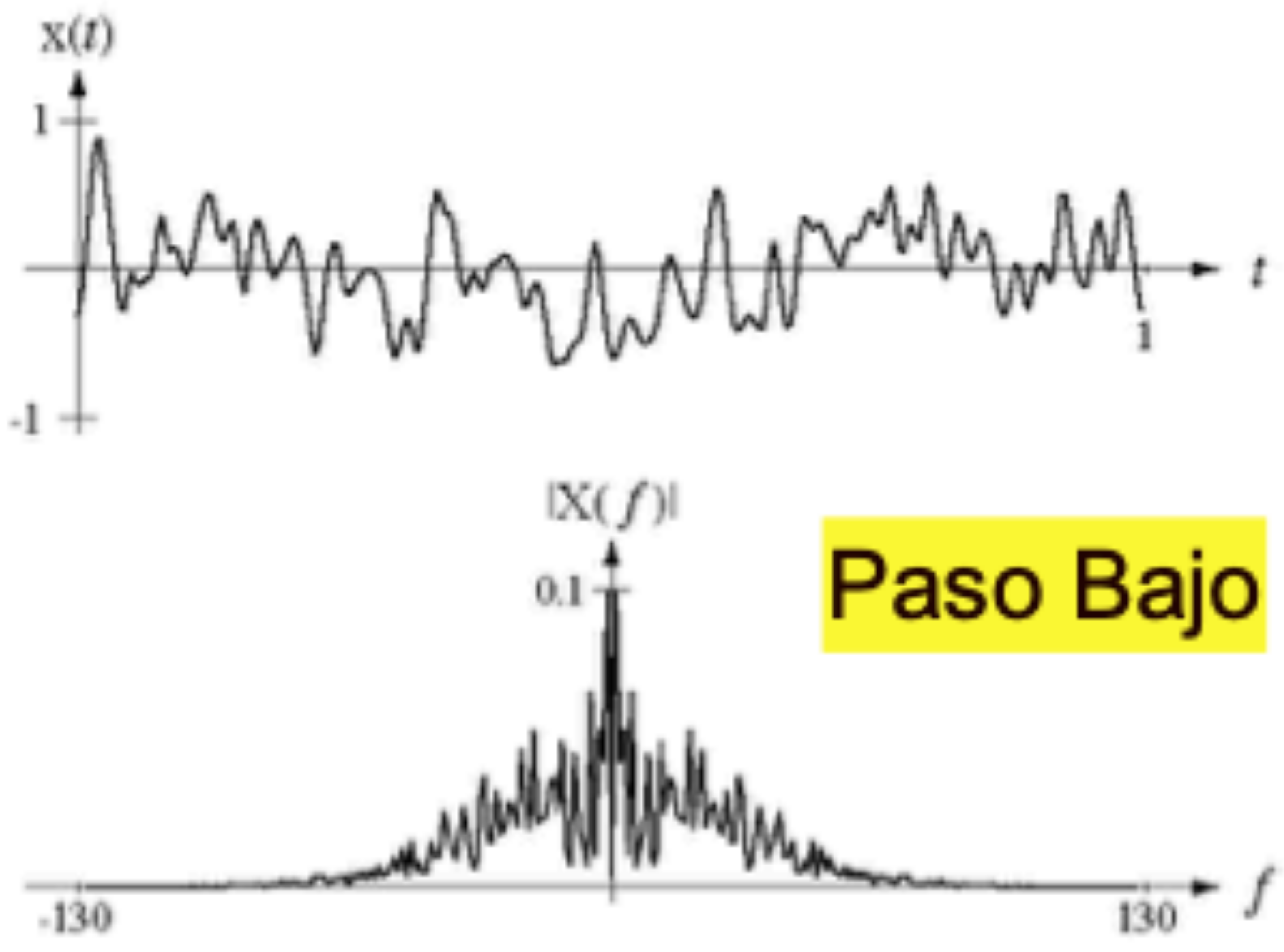
FINITE sum of periodic signals….
Namely x(t) can be expressed as a FINITE sum of periodic signals…

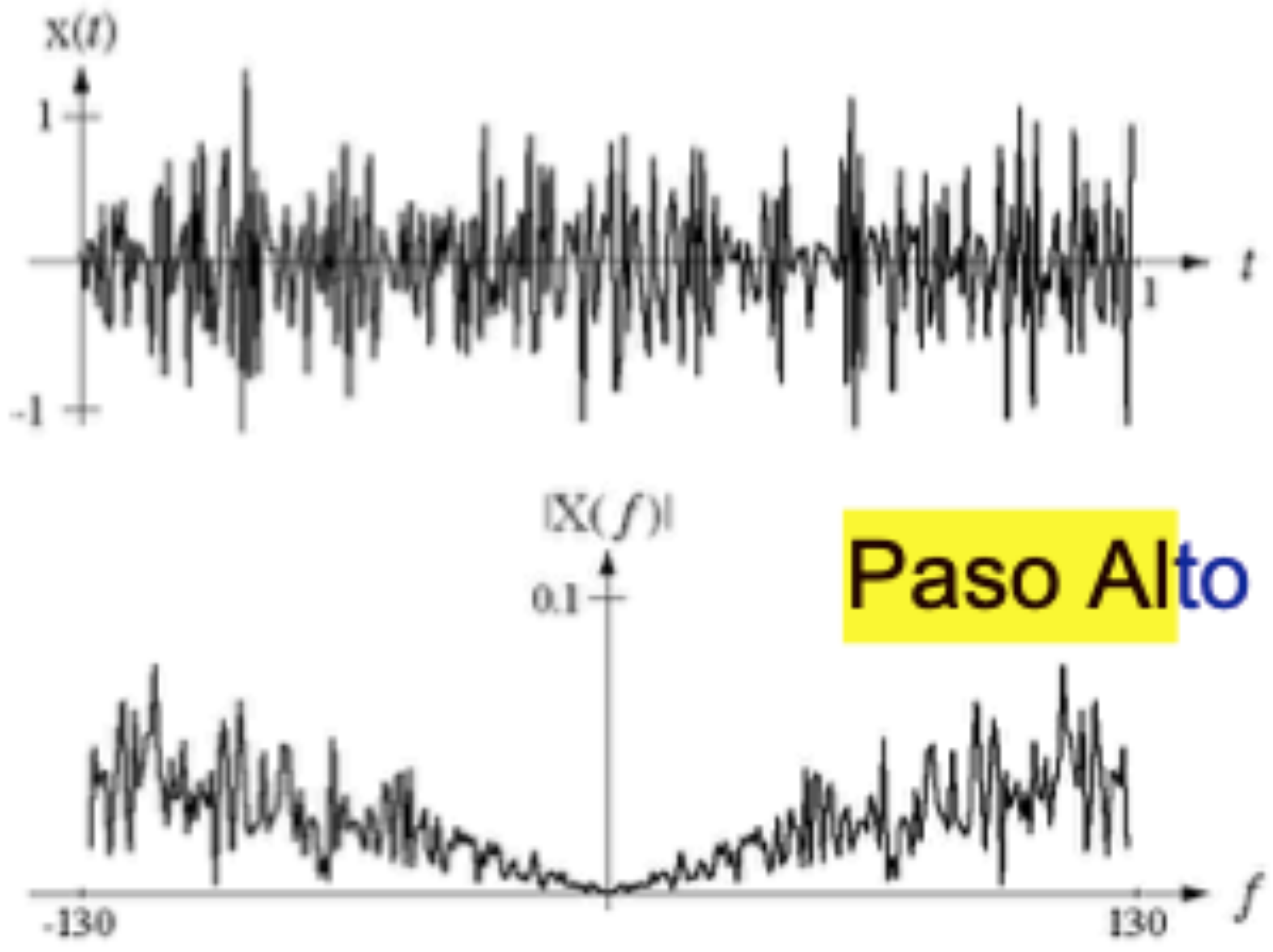
Frequencies *contained in* a signal?

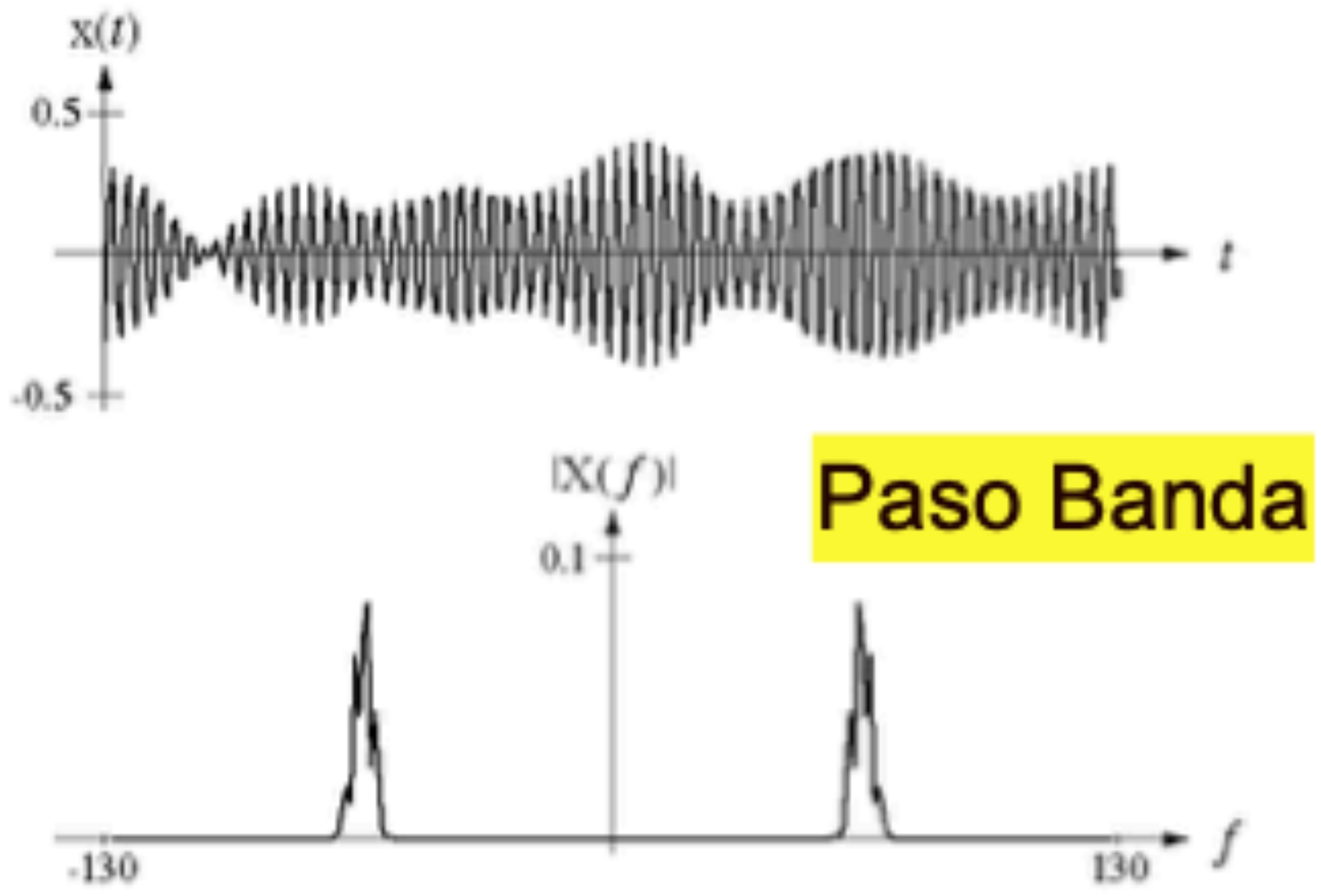
BUT generally, x(t) can be expressed as a INFINITE sum of periodic signals…



Contenido en frecuencia

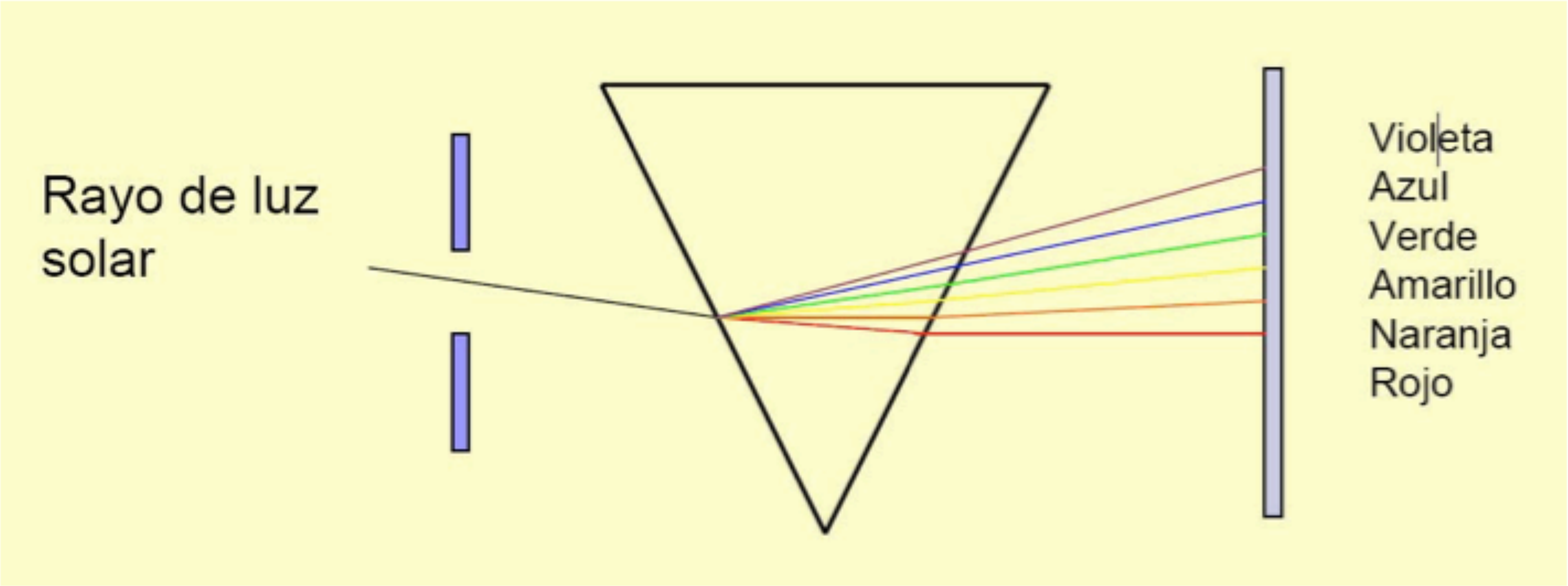






Waves… an example: electromagnetic !

- Ejemplo de descomposición de Fourier:
  - Luz blanca que atraviesa un prisma



Waves… an example: electromagnetic !

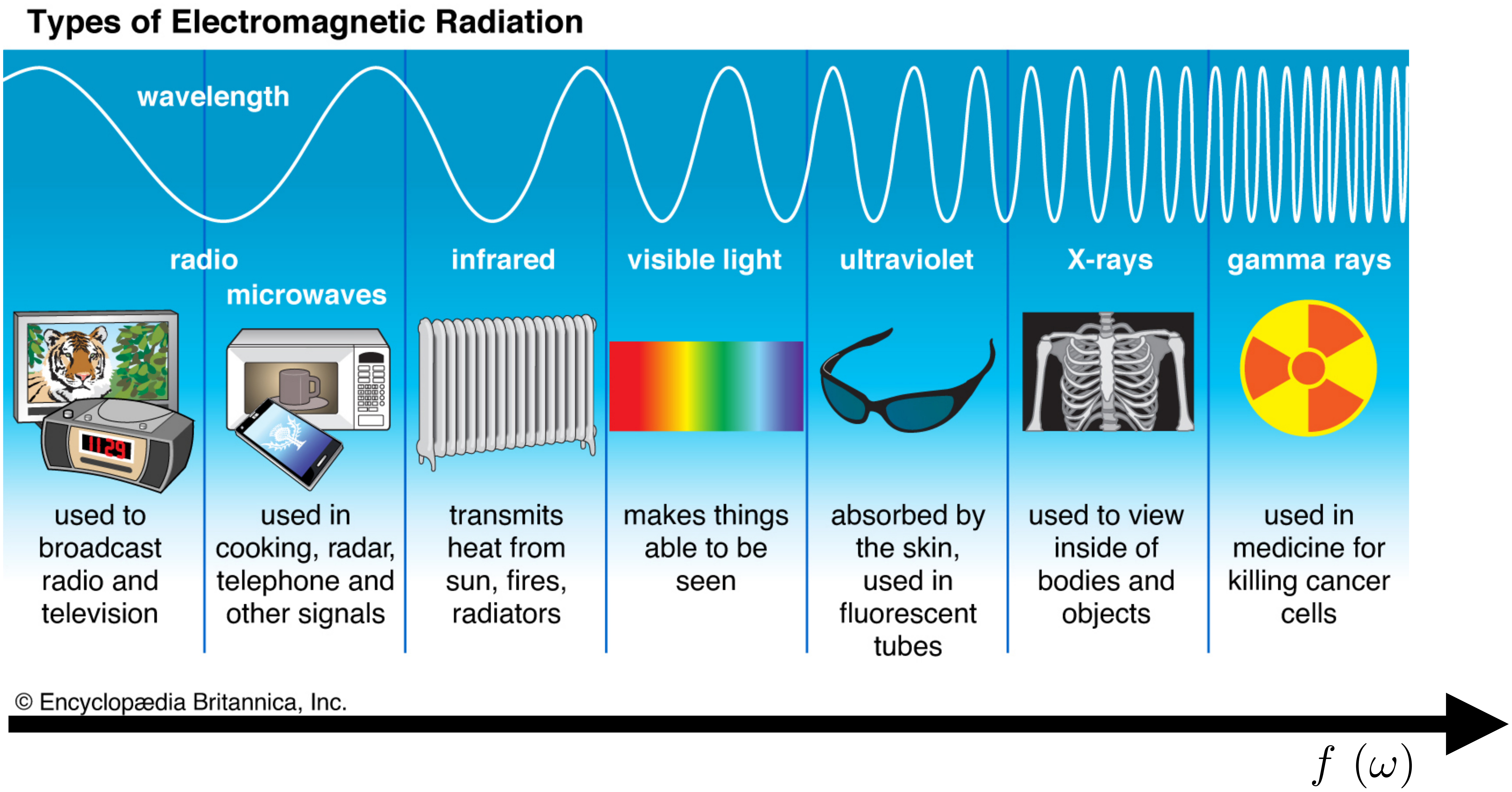
Electromagnetic waves are typically described by any of the following three physical properties: the frequency f, wavelength λ, or photon energy E. Frequencies observed in astronomy range from 2.4×10^{23} Hz (1 GeV gamma rays) down to the local plasma frequency of the ionized interstellar medium (~1 kHz). Wavelength is inversely proportional to the wave frequency,[5] so gamma rays have very short wavelengths that are fractions of the size of atoms, whereas wavelengths on the opposite end of the spectrum can be indefinitely long. Photon energy is directly proportional to the wave frequency, so gamma ray photons have the highest energy (around a billion electron volts), while radio wave photons have very low energy (around a femtoelectronvolt). These relations are illustrated by the following equations:

$$f = \frac{c}{\lambda}, \quad \text{or} \quad f = \frac{E}{h}, \quad \text{or} \quad E = \frac{hc}{\lambda},$$

where:

- $c = 299\,792\,458$ m/s is the speed of light in a vacuum
- $h = 6.626\,070\,15 \times 10^{-34}$ J·s $= 4.135\,667\,33(10) \times 10^{-15}$ eV·s is Planck's constant.

$$T = \frac{\lambda}{c}$$


Types of Electromagnetic Radiation
wavelength
radio
microwaves
infrared
visible light
ultraviolet
X-rays
gamma rays
used to broadcast radio and television
used in cooking, radar, telephone and other signals
transmits heat from sun, fires, radiators
makes things able to be seen
absorbed by the skin, used in fluorescent tubes
used to view inside of bodies and objects
used in medicine for killing cancer cells
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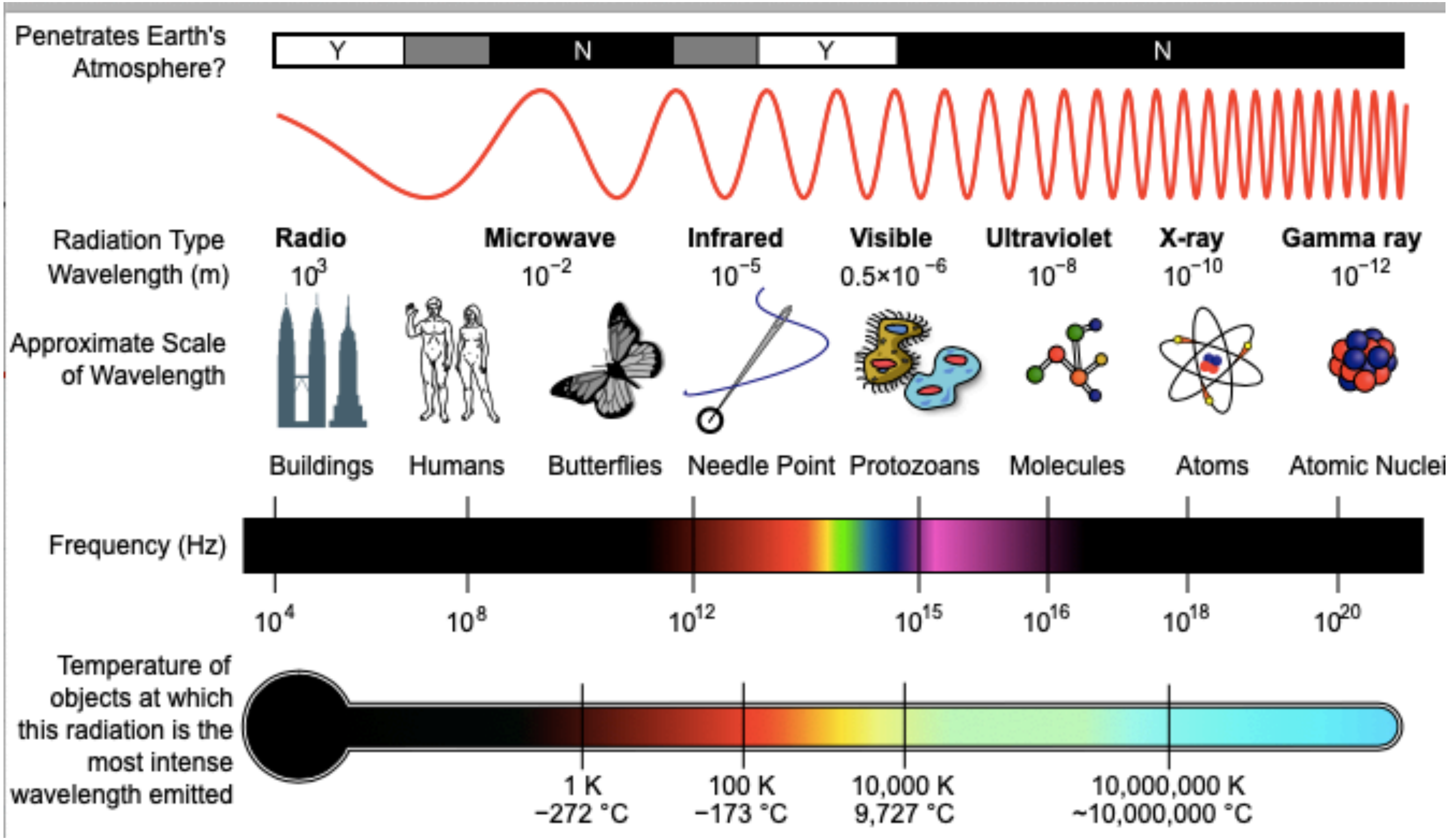
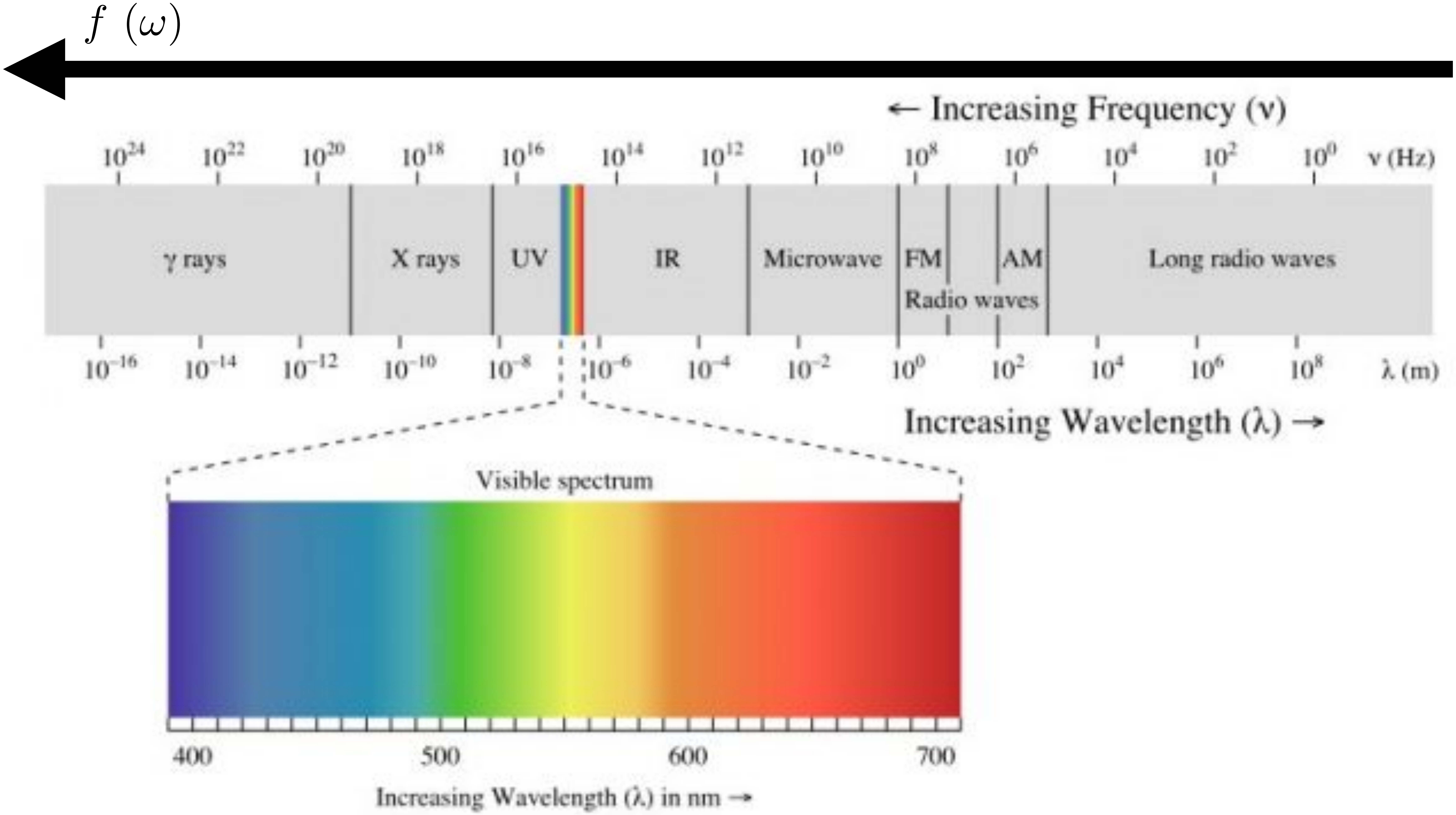
f (ω)

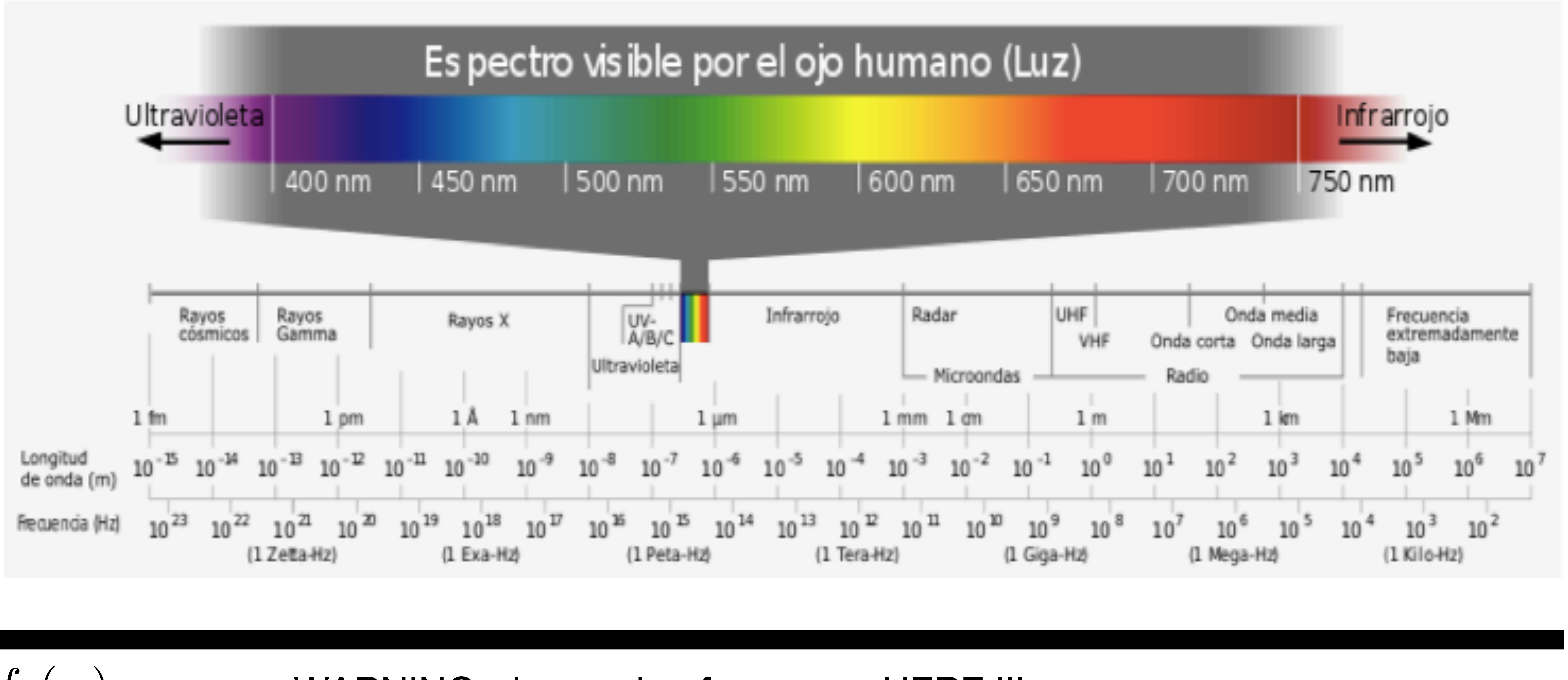


Penetrates Earth's Atmosphere?
Y
N
Y
N
Radiation Type
Wavelength (m)
Radio
10^3
Microwave
10^{-2}
Infrared
10^{-5}
Visible
0.5×10^{-6}
Ultraviolet
10^{-8}
X-ray
10^{-10}
Gamma ray
10^{-12}
Approximate Scale of Wavelength
Buildings
Humans
Butterflies
Needle Point
Protozoans
Molecules
Atoms
Atomic Nuclei
Frequency (Hz)
10^4
10^8
10^{12}
10^{15}
10^{16}
10^{18}
10^{20}
Temperature of objects at which this radiation is the most intense wavelength emitted
1 K
−272 °C
100 K
−173 °C
10,000 K
9,727 °C
10,000,000 K
~10,000,000 °C



$f\ (\omega)$
← Increasing Frequency (ν)
10^{24} 10^{22} 10^{20} 10^{18} 10^{16} 10^{14} 10^{12} 10^{10} 10^{8} 10^{6} 10^{4} 10^{2} 10^{0} ν (Hz)
γ rays
X rays
UV
IR
Microwave
FM
AM
Long radio waves
Radio waves
10^{-16} 10^{-14} 10^{-12} 10^{-10} 10^{-8} 10^{-6} 10^{-4} 10^{-2} 10^{0} 10^{2} 10^{4} 10^{6} 10^{8} λ (m)
Increasing Wavelength (λ) →
Visible spectrum
400
500
600
700
Increasing Wavelength (λ) in nm →

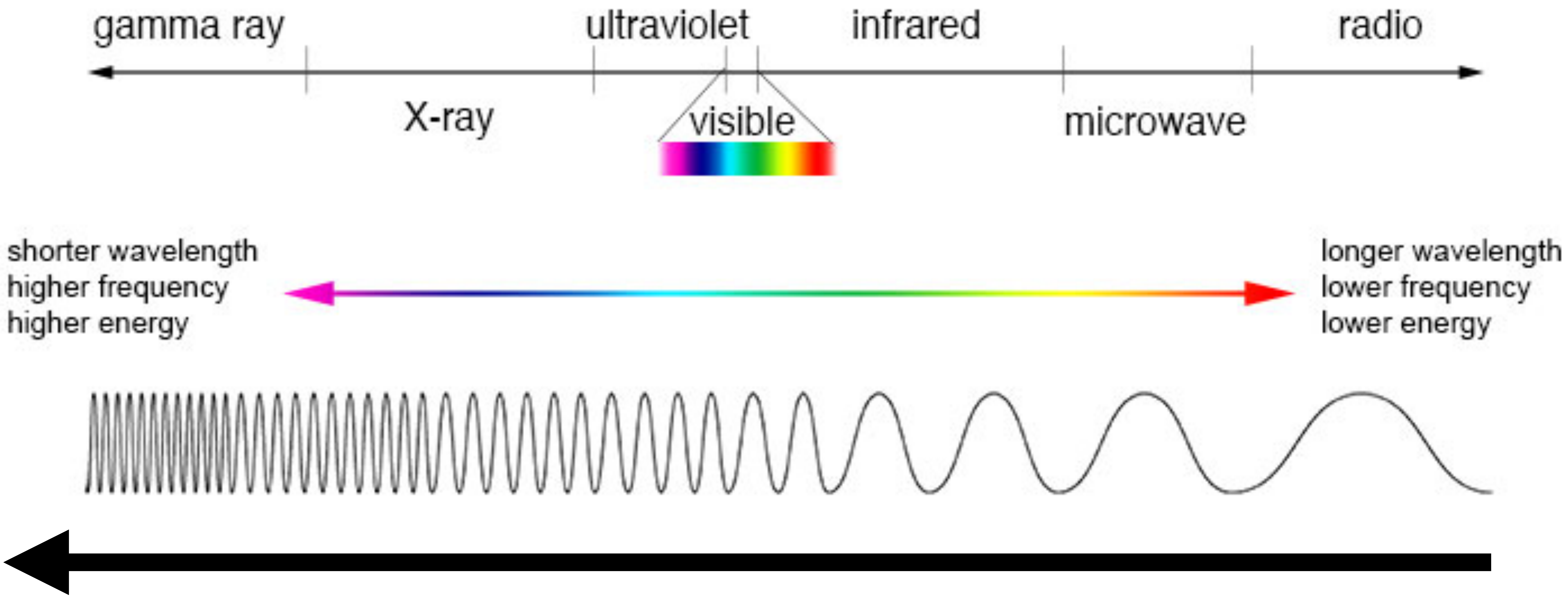
Espectro Electromagnético



$f \ (\omega)$

WARNING: decreasing frequency HERE !!!

Waves… an example: electromagnetic !



$f \ (\omega)$

WARNING: decreasing frequency HERE !!!

Questions?