

Topic 2.1 - Part 4: All the “transforms”

Señales y Sistemas

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Based also on Professor Óscar Barquero Perez, Andrés Martínez and José Luis Rojo's slides

Laplace and Standard Fourier Transforms

- Laplace and STANDARD Fourier transform for a generic signal $x(t)$ - Definition:

$$X(s) = \int_{-\infty}^{+\infty} x(t) e^{-s t} dt$$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j \omega t} dt$$

Existence of Laplace and Standard Fourier Transforms

- the integral should exist for each “s” and “w”:

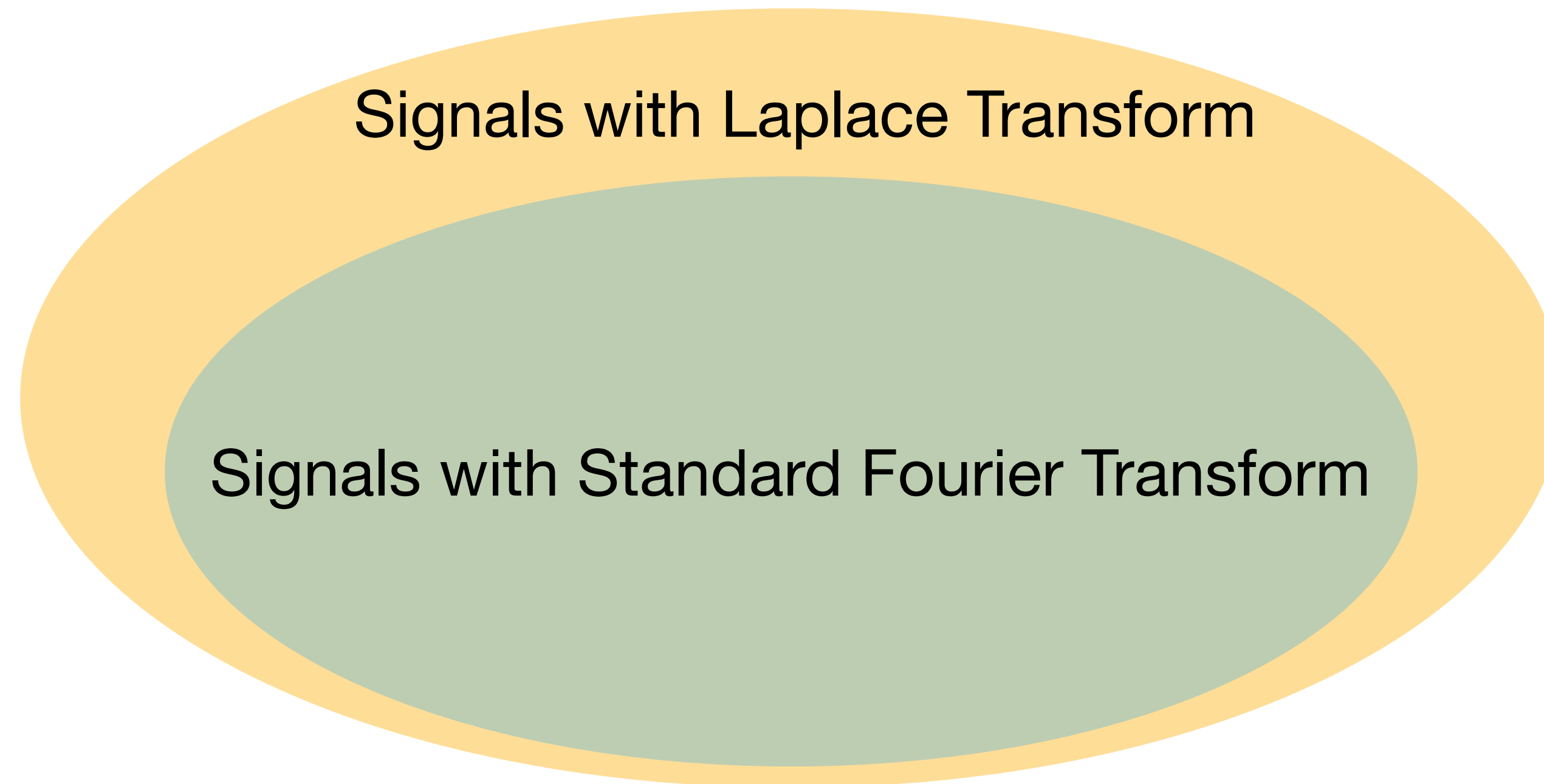
$$\int_{-\infty}^{+\infty} x(t) e^{-s t} dt$$

$$\int_{-\infty}^{+\infty} x(t) e^{-j \omega t} dt$$

- They must be FINITE and EXIST for all “s” and “w”

Laplace and Standard Fourier transforms

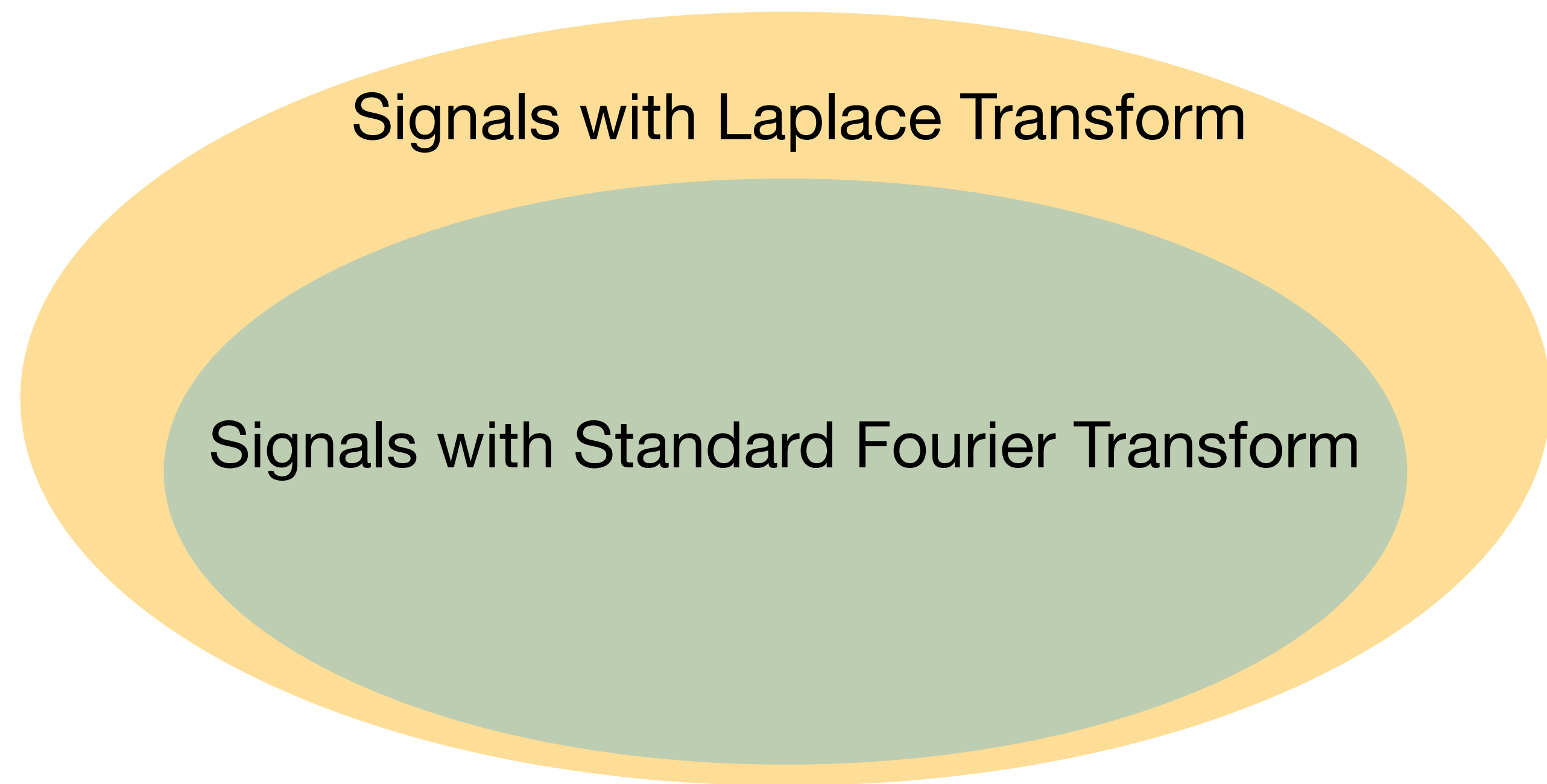
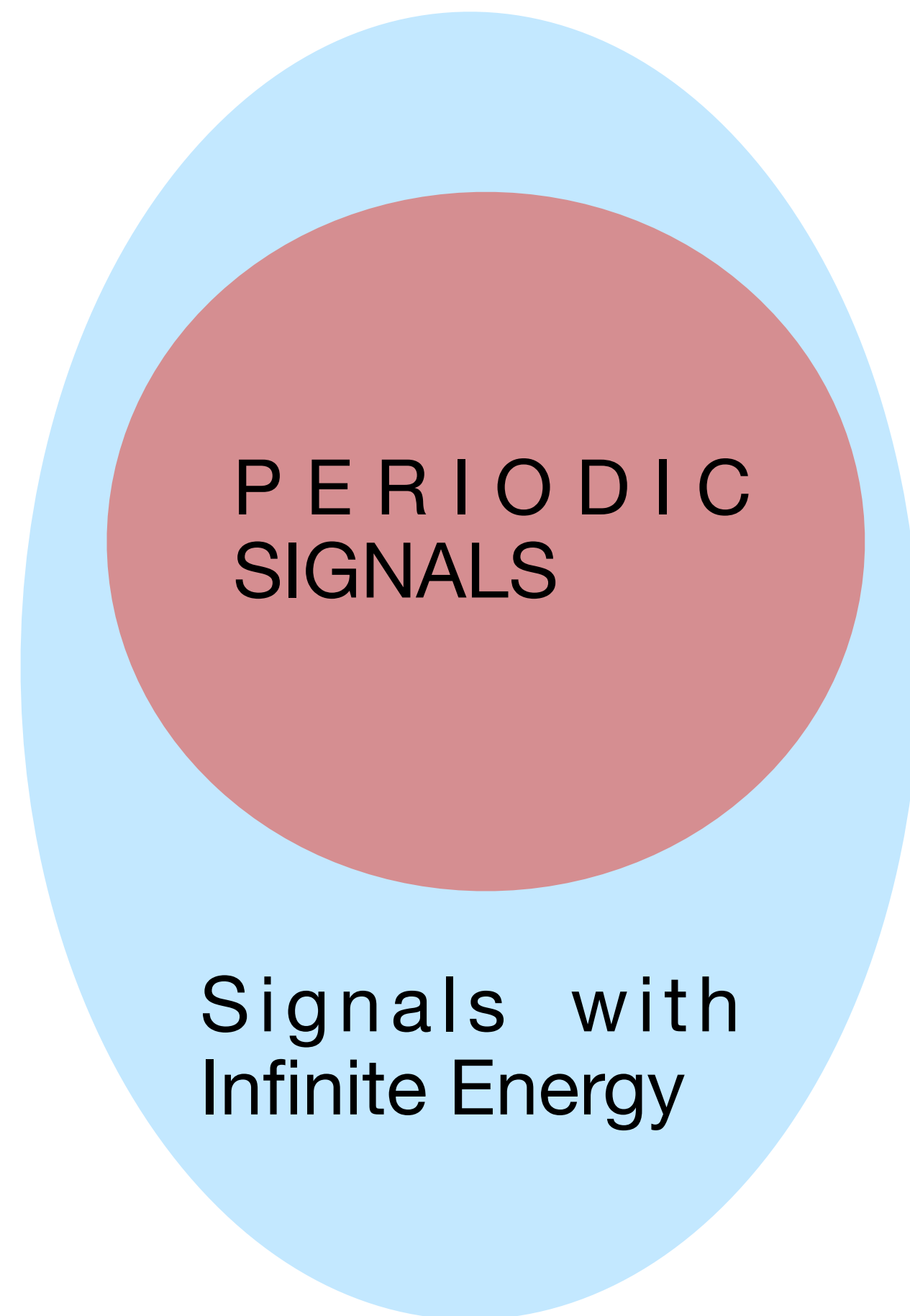
- **Some signals/functions do not have Laplace Transform.**
- **And even less signals/functions have Standard Fourier Transform....**



Laplace and Standard Fourier transforms

- **PERIODIC SIGNALS HAVE NOT LAPLACE/FOURIER**

Laplace permite la transformación de algunos señales de energía infinita también...



For a periodic signal? transformed domain?

- For periodic signals \implies Fourier series (FS)
- ... and the *Generalized Fourier Transform*
- (more generally, for some signals with infinite energy...)

For a periodic signal? transformed domain?

- **For periodic signals ==> Fourier series (FS)**
- *... and the **Generalized Fourier Transform (GFT)***
- *but it is the “same” that of the Fourier Series for periodic signals....*
- all the information is contained in the FS (with GFT nothing new, for periodic signals)

For a periodic signal? transformed domain?

- **Historically, the Fourier Series was the first tool introduced...**

Transformations for signals in continuous time

For Periodic signals

For non-periodic signals

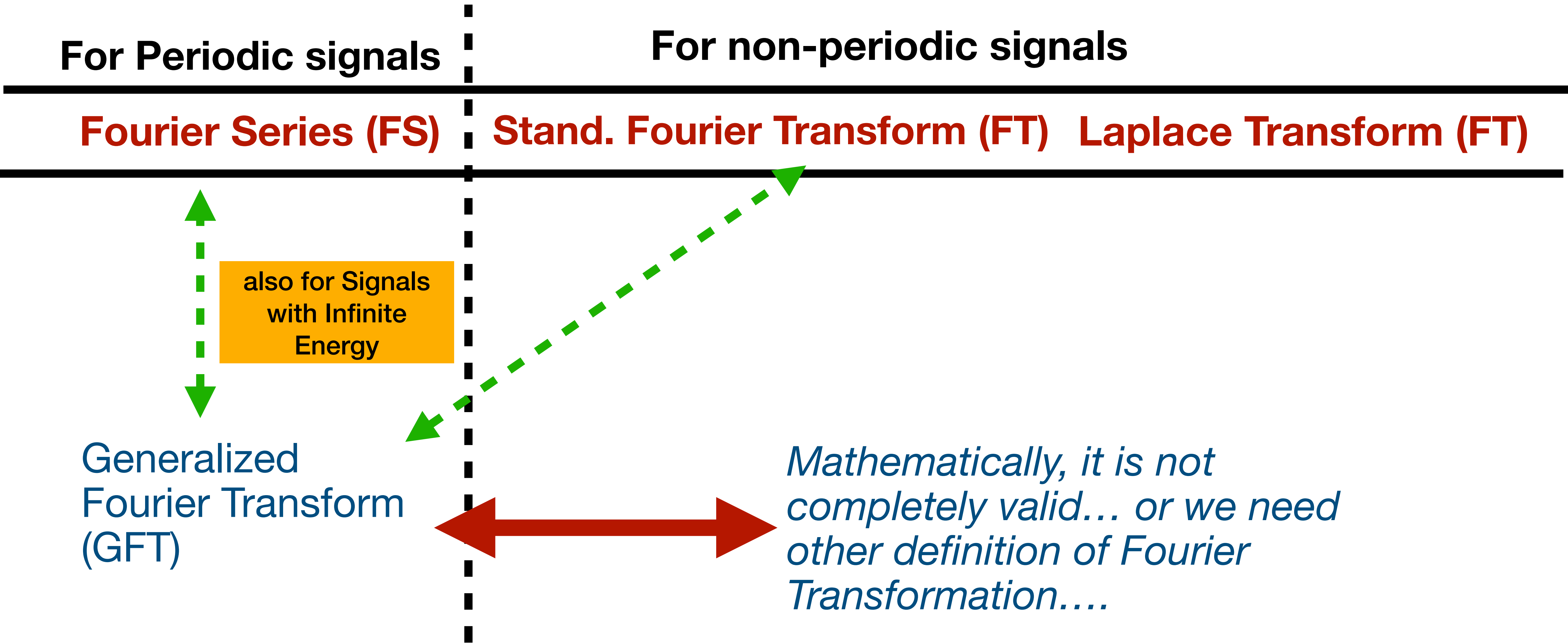
Fourier Series (FS)

Stand. Fourier Transform (FT) **Laplace Transform (FT)**

also for Signals
with Infinite
Energy

Generalized
Fourier Transform
(GFT)

*Mathematically, it is not
completely valid... or we need
other definition of Fourier
Transformation....*



Transformations for signals in discrete time

For Periodic signals

For non-periodic signals

Fourier Series

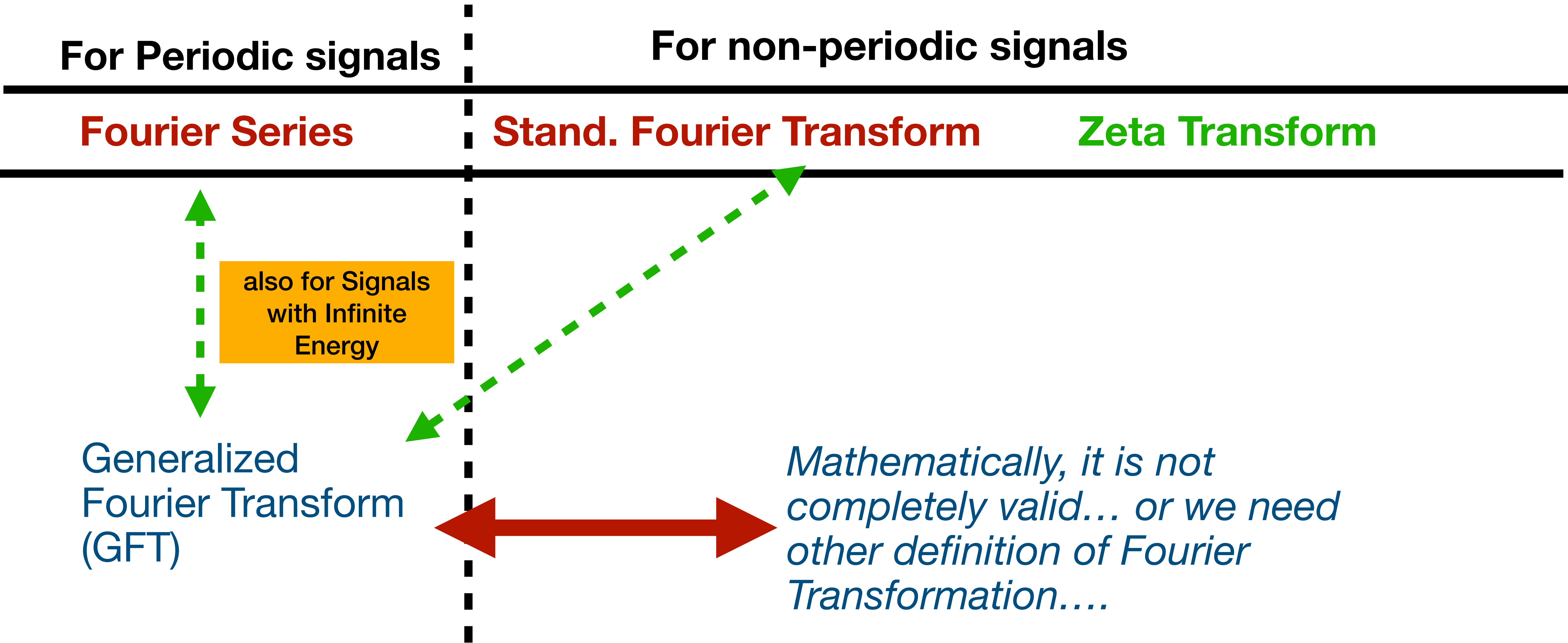
Stand. Fourier Transform

Zeta Transform

also for Signals
with Infinite
Energy

Generalized
Fourier Transform
(GFT)

*Mathematically, it is not
completely valid... or we need
other definition of Fourier
Transformation....*



Next?

- **the Fourier Series (FS) for signals in continuous time**

Questions?