Topic 2.2 - Part 3: Fourier Series

Linear systems and circuit applications

Señales y Sistemas

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Based also on Professor Óscar Barquero Perez, Andrés Martínez and José Luis Rojo's slides

Fourier Series

- Definition of the Fourier series (**synthesis equation**):

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

- **a_k: complex coefficients**
- **k integer variable**

Fourier Series (analysis equation)

- Frequency information is **contained in the a_k's**

$$a_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$$

- **INTEGRAL IN A PERIOD - from 0 to To or -To/2 to To/2, for instance.**
- a_k: complex coefficients
- k integer variable

“intuitive” derivation of the a_k

- Como llegar a los a_k? (derivación intuitiva, no formal) we show an “intuitive” derivation of the coefficients a_k:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$x(t)e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} e^{-jn\omega_0 t}$$

“intuitive” derivation of the a_k

$$x(t)e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} e^{-jn\omega_0 t}$$

$$\int_{T_0} x(t)e^{-jn\omega_0 t}dt = \int_{T_0} \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} e^{-jn\omega_0 t}dt$$

$$\int_{T_0} x(t)e^{-jn\omega_0 t}dt = \sum_{k=-\infty}^{\infty} a_k \left(\int_{T_0} e^{j(k-n)\omega_0 t}dt\right)$$

“intuitive” derivation of the a_k

$$\int_{T_0} x(t)e^{-jn\omega_0 t}dt = \sum_{k=-\infty}^{\infty} a_k \left(\int_{T_0} e^{j(k-n)\omega_0 t}dt\right)$$

$$\int_0^{T_0} x(t)e^{-jn\omega_0 t}dt = \sum_{k=-\infty}^{\infty} a_k \left(\int_0^{T_0} e^{j(k-n)\omega_0 t}dt\right)$$

We focus on it

“intuitive” derivation of the a_k

$$\int_0^{T_0} e^{j(k-n)\omega_0 t} dt = \int_0^{T_0} \cos([k-n]\omega_0 t) dt + j \int_0^{T_0} \sin([k-n]\omega_0 t) dt$$

We are lucky since the integration is in a finite interval…

after some “looks” and easy derivations, we can arrive to:

$$\int_0^{T_0} e^{j(k-n)\omega_0 t} dt = \begin{cases} T_0, & k = n \\ 0, & k \neq n \end{cases}$$

“intuitive” derivation of the a_k

$$\int_0^{T_0} e^{j(k-n)\omega_0 t} dt = \begin{cases} T_0, & k = n \\ 0, & k \neq n \end{cases}$$

$$\int_0^{T_0} e^{j(k-n)\omega_0 t} dt = T_0 \delta[n-k]$$

“intuitive” derivation of the a_k

$$\int_0^{T_0} x(t)e^{-jn\omega_0 t}dt = \sum_{k=-\infty}^{\infty} a_k \left(\int_0^{T_0} e^{j(k-n)\omega_0 t}dt \right)$$

$$\int_0^{T_0} x(t)e^{-jn\omega_0 t}dt = \sum_{k=-\infty}^{\infty} a_k \left(T_0\delta[n-k] \right)$$

$$\int_0^{T_0} x(t)e^{-jn\omega_0 t}dt = T_0 a_n$$

“intuitive” derivation of the a_k

$$\int_0^{T_0} x(t)e^{-jn\omega_0 t}dt = T_0 a_n$$

$$a_n = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jn\omega_0 t}dt$$

Questions?